Inventory and pricing decisions for a dual-channel supply chain with deteriorating products

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Abstract Dual-channel supply chain structure, i.e., a traditional retail channel added by an online direct channel, is widely adopted by a lot of firms, including some companies selling deteriorating products (e.g. fruits, vegetables and meats, etc.). However, few papers in literature consider deterioration property of products in dual-channel business models. In this paper, a single-retailer-single-vendor dual-channel supply chain model is studied, in which the vendor sells deteriorating products through its direct online channel and the indirect retail channel. In addition to quantity deterioration, quality of the products also drops with time and affects the demand rate in the retail channel. The pricing decisions and the inventory decisions for the two firms are simultaneously studied. Models of centralized (i.e., the two firms make decisions jointly) and decentralized (i.e., the two firms make decisions separately, vendor as the Stackelberg leader) problems are established. Proper algorithms are proposed to obtain the optimal decisions of prices, ordering frequencies and ordering quantities. The results suggest that decentralization of the supply chain not only erodes the two firms' profit, but also incurs higher wastes comparing to that under centralization. However, a revenue sharing and two part tariff contract can coordinate the supply chain. Under utilizing the contract, each firm's profit is improved and the total waste rate of the supply chain is reduced. It is also shown that the contract is more efficient for both firms under higher product deterioration rate. Besides, the contract is more efficient for the retailer, while less efficient for the vendor under higher quality dropping rate. In the model extension, online channel delivery time is assumed to be endogenous and linked to demands in both channels. The results show that products' deterioration rate and quality dropping rate have significant impacts to the firms' delivery time decisions, as well as the pricing and inventory decisions.

Keywords Dual channel supply chain · Product deterioration · Game theory · Pricing · Inventory

1 Introduction

With the fast development of Internet and information technology, customers' purchasing behaviors have been changed a lot, which pushes more and more firms in various industries to establish direct selling channels in addition to the traditional 'Brick-and-Mortar' channels. By establishing direct channels, companies can often benefit from the expanded market coverage (Chen et al., 2012), the enhanced control power over retail price (Chiang et al., 2003), etc. According to a survey, about 42% of the top PC manufacturers (like

Dell, Sony, Compaq, Lenovo, etc.) are selling through their own direct channels (Wilder, 1999; Tsay & Agrawal, 2004). Today, in the fresh food industry, with developed preservation and logistic technologies, many companies are selling their goods to customers through the direct online channel. For the grocery giant Wal-Mart, fresh foods are sold through Chinese e-commerce partner Yihaodian in Shanghai and Beijing [1]. Also, Harry and David, as one of Internet Retailer 500 and America's leading gourmet gifting companies, sells fresh foods through both direct and retail channels. Nowadays, in China, more and more customers choose to buy fruits on e-commerce websites, including Alibaba, Taobao and Tmall, etc. Khuntonthong et al. (2013) demonstrated that the development of e-commerce techniques gives farmers more opportunities to benefit from perishable agricultural foods.

As fresh-selling through dual-channel is becoming more and more popular, the research on the management of dual-selling for deteriorating products is required and urgent. However, most of the previous studies on dual-channel problems concentrate on the single period pricing problems. Since the product deterioration is a time linked phenomenon, most of their models cannot characterize product deterioration appropriately. A common way to study product deterioration is to use the EOQ models.[2] So, in this paper, in addition to the pricing decisions, inventory decisions are considered for deterioration items. In the centralized model, the vendor and the retailer are vertically integrated. They make decisions together on both the prices of the two channels and the inventory for the whole supply chain. In addition to the centralized model, a decentralized model is studied, in which, the vendor and the retailer competes vertically and horizontally on pricing and ordering decisions. In the game, the vendor is the Stackelberg leader and the retailer is the follower. Also, a revenue sharing and two part tariff contract is proposed to coordinate the supply chain, which can not only raise both firms' profits, but also reduce the total waste of the whole supply chain. In the model extension, a more realistic model is studied, in which the direct channel delivery time is endogenous. The main research questions in this paper can be summarized as:

(1) How to obtain the optimal decisions under both centralized and decentralized supply chains?

(2) How does the optimal decisions change with the critical parameters (e.g., deterioration rate, competition intensity, etc.)?

(3) Can the revenue sharing and two part tariff contract coordinate the supply chain? If so, how to determine the optimal contract parameters?

(4) How to decide the optimal delivery time when it is endogenous?

The rest of this paper is organized as follows. Section 2 is the literature review. Section 3 introduces the notations and assumptions through the paper. Then, the centralized and decentralized models are formulated. To solve the models, two algorithms are also proposed. In section 4, numerical examples and sensitivity analysis are presented, along with some important results and interesting managerial implications. In section 5, a revenue sharing and two part tariff contract is proposed to coordinate the supply chain. Section 6 is the model extension. In the last section, conclusions for the paper are presented, and some future research topics are suggested.

[1] Russel, J. (Jul 22, 2015) Walmart Takes Full Control Of Yihaodian, Its Online Retail Business In China. https://techcrunch.com/2015/07/22/walmart-buys-out-its-chinese-store-yihaodian/. Accessed on November 7, 2016

[2] The EOQ theory enables people to consider the transaction costs (e.g. deterioration cost, transportation cost, inventory holding cost, ordering cost) which have great impacts to pricing and ordering decisions for supply chain members. According to Moss et al. (2003), transaction cost (including deterioration cost, transportation cost, inventory holding cost, ordering cost, etc.) is an important factor for the application of e-commerce business structures. The ignorance of the transaction cost may result in non-optimal decisions.

2 Literature review

This paper is closely linked to two streams: (1) dual channel supply chain models (2) EOQ/EPQ models with product deterioration.

One stream of literature is about the research of dual-channel business models. In recent couple of years, dual-channel business model has been deeply studied by researchers on supply chain management and marketing. Chiang et al. (2003) argued that the motivation of adding a direct channel is to reduce the double marginalization effects. Yan & Pei (2009) pointed out that establishing a direct channel is a useful tool for the manufacture to motivate the retailer's service level improvement, and to enhance the efficiency of the total supply chain. In addition to the pricing decisions, researchers consider about other important and realistic factors, such as direct channel delivery lead time, service level for both channels, demand disruption, asymmetric information, etc. Hua et al. (2010) showed that in a dual-channel supply, lead time has strong effect on both parties' pricing and quantity decisions. Xu et al. (2012) extended Chiang et al. (2003) by treating the delivery time length of the online channel as a decision variable. They also showed that lead time decision has effects to the manufacturer's channel selections. Yang et al. (2017) studied a dual-channel Newsvendor model with lead time linked demand and customers' switching behaviors when shortage occurs. Chen et al. (2017) studied the quality decisions in addition to the pricing decisions in a dual-channel supply chain. They showed that adding another channel can improve the product quality and the supply chain performance. Xiao et al. (2014) investigated the product variety design and pricing decisions for a two level supply chain in a circular spatial market under manufacturer-lead and retailer-lead Stackelberg gaming. They found that the motivation for the manufacturer to use dual channels decreases with the unit production cost, while increases with the marginal cost of variety design, the retailer's marginal selling cost, and the customers' fit cost. Cai et al. (2009) and Chen et al. (2012) did excellent study on channel selection policies under different values of selling costs, potential market share and competition intensity. Dumrongsiri et al. (2008) found that a higher retail service level or customer service sensitivity can benefit both parties when demand is price and service dependent. Li & Li (2016) studied a dual-channel supply chain with retailer's service investment and fairness concerns. Mukhopadhyay et al. (2008) studied a dual-channel model with a value adding retailer who has private information of the value adding costs. They found that a lower cost for the retailer can induce the information sharing throughout the supply chain. Dan et al. (2012) argued that the market share and the customers' loyalty of retail channel has great influence to the pricing and service decisions. Liu et al. (2015) studied the manufacturer's and retailer's risk aversion behaviors under asymmetric information to the optimal decisions. Yan et al. (2016) studied the optimal pricing in a dual-channel supply chain with a dominant retailer and two manufacturers, in which the manufacturers lie about their cost information. Li et al. (2016a) studied the pricing and coordination problems in a dual-channel supply chain with a risk-averse retailer. Liu et al. (2010) demonstrated that when selling cost information is private, centralized decision is not always better than the decentralized decision with a feasible contract if the retailer has lower selling cost. Chiang & Monahan (2005) found that when demand is uncertain and demand can be transferred from one channel to another, dual-channel is better than either the pure retail channel or the pure direct channel. Yu et al. (2016) studied the impacts of the e-tailer's drop-shipping decisions to the manufacturer's distribution channel strategies. He et al. (2014) studied the transshipment strategies between the e-store and the retailer under demand uncertainties in both channels. Xiao & Shi (2016) studied the optimal pricing decisions in the presence of supply shortage under different supply priority of each channel. Yue & Liu (2006) found that for uncertain demand, the gap of demand estimated by the manufacture and that by the retailer has significant impacts on the benefit of both parties. Huang et al. (2012) found that when demand is disrupted and the demand disruption level falls in an interval, the optimal production quantity decision is robust. Choi et al. (2013) studied the decisions under different power structures of the dual-channel supply chain when products can be recycled. Matsui (2015) studied the channel strategies (single channel or dual channel) of two competing manufacturers. They showed that, the symmetry of the two manufacturers can result in asymmetry equilibrium. Matsui (2017) studied the optimal decision sequence of the direct price and wholesale price in a dual-channel supply chain. They showed that the manufacturer should announce its direct price before or upon the wholesale price. Lu & Liu (2015) studied the entry of an external e-commerce channel to the manufacturer's channel selections. Li et al. (2016b) studied the pricing, greenness and manufacturer's channel selections in a dual channel green supply chain. He et al. (2016) studied the carbon emissions in a dual-channel supply chain in the presence of customers' free riding behaviors. Ji et al. (2017) studied manufacturer's carbon emission reduction efforts, pricing and channel selection decisions in a dual-channel supply chain. Takahashi et al. (2011) studied the inventory decisions in a two echelon dual-channel supply considering the manufacturer's and retailer's stock setup and delivery decisions. Rodriguez & Aydin (2015) studied the pricing and assortment decisions in a dual-channel supply chain. Chen (2015) studied the cooperative advertising strategies in a dual-channel supply chain. Xie et al. (2017) coordinated the dual-channel supply chain in the presence of cooperative advertising with a revenue sharing contract. Batarfi et al. (2016) studied a centralized decision model with price competition between the two channels. In addition to the pricing decisions, they also studied the inventory decisions. Their research is closely related to this paper. However, this paper is different from Batarfi et al. (2016) in two aspects. Firstly, product deterioration is considered in this paper, which was seldom considered in previous research, including Batarfi et al. (2016). Secondly, in Batarfi et al. (2016), they studied an integrated model. However, this paper also studies a decentralized model, in which both horizontal and vertical competitions between the vendor and the retailer are considered.

In addition to the competition problems, researchers also studied the coordination of the decentralized supply chain with revenue sharing contracts (Yan, 2008; Cai, 2010; Xie et al., 2017), two-part tariff and profit sharing contracts (Chen et al., 2012), price discount contracts (Cai et al., 2009) under different situations. In this paper, a revenue sharing and two part tariff contract is proposed to coordinate the dual channel supply chain with product deterioration. Summary of the related literature on dual-channel supply chain is shown in **Tab.1**. To our best knowledge, this paper is the first that consider product deterioration, inventory decision, supply chain coordination simultaneously in a dual-channel supply chain.

Another related stream of literature is EOQ/EPQ models for deteriorating products. Summary of the related literature on dual-channel supply chain is shown in **Tab.2.** According to Shah et al. (2013), deterioration is defined as decay, change or spoilage so that the items are not in its initial conditions. There are two categories of deterioration items. The first category refers to the items that become decayed, damaged or expired with time, e.g., meat, vegetables, fruits, medicine, etc. The second category is the items that lose part or total value with time, e.g., computer chips, mobile phones, fashion and seasonal products, etc. Both kinds of items have short life cycles and after a period of existence in market, the items lose the original economical value due to the drop of consumer preference, product quality, etc. Ghare & Schrader (1963)

Tab. 1 Related literature on dual-channel supply chain

Research paper	Pricing	Inventory policy	SC coordination	Others
Chiang et al. (2003)	√			
Chen et al. (2012)	√		√	
Choi et al. (2013)	√		√	
Cai et al. (2009)	√			
Chiang & Monahan (2005)	√	√		
Chen (2015)	√			Cooperative advertising
Xie et al. (2017)	√		√	Cooperation advertising
Yan & Pei (2009)	√			Retailer service
Mukhopadhyay et al. (2008)	√			Retailer service, information sharing
Dan et al. (2012)	√			Retailer service
Li & Li (2016)	√			Retail service
Hua et al. (2010)	√			Lead time
Xu et al. (2012)	√			Lead time
Yang et al. (2017)	√			Lead time, stochastic demand
Xiao et al. (2014)	√			Product variety
Dumrongsiri et al. (2008)	√			Stochastic demand
Xiao & Shi (2016)	√			Uncertain supply, channel priority
Li et al. (2016a)	√		√	Risk aversion
Liu et al. (2015)	√			Risk aversion
Yue & Liu (2006)	√			Information sharing
Huang et al. (2012)	√			Demand disruption
He et al. (2014)	√			Transshipment, stochastic demand
Liu et al. (2010)	√			Stochastic demand, information sharing
Chen et al. (2017)	√			Quality decision
Yu et al. (2016)	√			Drop shipping strategy
Matsui (2015)	√			Competition of two manufacturers
Matsui (2017)	√			Pricing sequence decision
Lu & Liu (2015)	√			External e-commerce competition
Yan et al. (2016)	√			Retailer dominated SC
Li et al. (2016b)	√			Product greenness
He et al. (2016)	√			Product greenness, free-riding behavior
Ji et al. (2017)	√			Product greenness
Takahashi et al. (2011)		√		Delivery decisions
Rodriguez & Aydin (2015)	√	√		Assortment planning
Batarfi et al. (2016)	√	√	√	
This paper	√	√	√	Product deterioration

Tab. 2 Related literature on deteriorating inventory models

Research paper	Pricing	Deterioration	SC level	Others
Ghare & Schrader (1963)		Quantity	One	
Sarker et al. (1997)		Quantity	One	Inventory-level dependent demand
Giri et al. (2003)		Quantity	One	Ramp type demand
Sana et al. (2004)		Quantity	One	Demand shortage
Dye et al. (2006)		Quantity	One	Demand backlogging
Chen & Chen (2007)		Quantity	Three	Multi-item
Lo et al. (2007)		Quantity	Two	Integrated decision
Skouri et al. (2009)		Quantity	One	Weibull deterioration rate
Thangam & Uthayakumar (2009)	√	Quantity	Two	Trade credit
Lin et al. (2009)	√	Quantity	Two	Cooperative decision
Lin et al. (2010)	√	Quantity	Two	Cooperative decision
Hsu et al. (2010)		Quantity	One	Preservation investment
Liang & Zhou (2011)		Quantity	One	Two warehouse, trade credit
Wang et al. (2011)		Quantity	Three	Time dependent deterioration rate
Sarkar (2011)		Quantity	One	Trade credit, demand shortage
Mahata (2012)		Quantity	One	Trade credit
Sarkar (2012a)		Quantity	One	Trade credit
Dye & Hsieh (2012)		Quantity	One	Preservation investment
Dye & Hsieh (2013)		Quantity	One	Preservation investment
Sarkar et al. (2013)	√	Quantity	One	Component cost
Sarkar & Sarkar (2013)		Quantity	One	Probabilistic deterioration
Shah et al. (2013)	√	Quantity	One	Non-instantaneous deterioration
Sarkar (2013)		Quantity	Two	Probabilistic deterioration
Qin et al. (2014)	√	Quantity and quality	One	Fresh produce
Chauhan & Singh (2015)		Quantity	One	Cash flow discount
Sarkar et al. (2015)		Quantity	One	Fixed lifetime
Zhang et al. (2015)	√	Quantity	Two	Preservation investment
This paper	√	Quantity and quality	Two	Dual channel supply chain

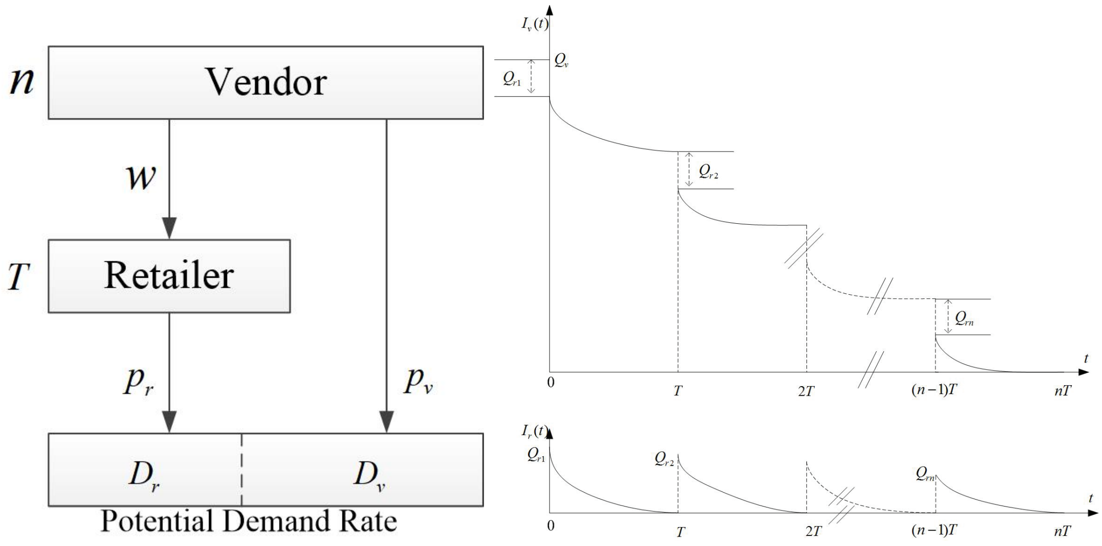
first proposed an exponentially decaying inventory model. Based on their work, people had done a lot on the EOQ problems for deterioration products. In this research area, different settings of critical factors, e.g., demand rate, deterioration rate, pricing strategies, etc., have significant impacts on the formulation of the models, and the associated solutions and results. Firstly, for demand rate, it can either be a constant parameter (Mahata, 2012) or be a time dependent parameter (Giri et al., 2003; Wang et al., 2011; Dye et al., 2006; Sarkar, 2012a). Also, demand can be backlogged (Dye et al., 2006), inventory level linked (Burwell et al., 1997; Sarker et al., 1997) or price sensitive (Shah et al., 2013; Dye & Hsieh, 2012; Liang & Zhou, 2011). Secondly, for deterioration rate, it can be a constant parameter (Sana et al., 2004; Thangam & Uthayakumar, 2009; Liang & Zhou, 2011; Sarkar, 2013; Sarkar et al., 2013), a time linked parameter (Skouri et al., 2009; Sarkar et al., 2015; Sarkar, 2011) , preservation investment linked parameter (Hsu et al., 2010; Dye & Hsieh, 2013) or a stochastic parameter (Sarkar & Sarkar, 2013; Sarkar, 2013, 2012b).

The above research only consider the single stage inventory problems. Some people studied the problems in multi level supply chains. Lee & Moon (2006) proposed a basic three level producer-vendor-buyer model. Wang et al. (2011) extended Lee & Moon (2006) by assuming that products suffers from time linked deterioration rate. Besides, many researchers did a lot of work on integrated inventory and/or pricing decisions (Lo et al., 2007; Chen & Chen, 2007; Noh et al., 2016; Sarkar et al., 2016), Stackelberg gaming problems (Song & Zhao, 2010), and cooperation strategies (Lin et al., 2009, 2010; Sarkar, 2016) in multi-level supply chains.

To the best of our knowledge, models and analysis in this paper is novel and different from the extent papers on dual-channel supply chain. The main contributions of this paper are quadruple. Firstly, product deterioration is considered in a dual channel supply chain, which is rarely mentioned in previous literature on dual channel supply chain. Moreover, two kinds of product deterioration is considered, i.e., the quantity deterioration and the quality deterioration. The quantity deterioration affects the firms' inventories, whereas the quality deterioration affects customers' choices and demand rate. The problem is modeled over an infinite time horizon, which enables us to better characterize the products' deterioration property. Secondly, pricing and inventory decisions are studied simultaneously in the dual channel supply chain, which is seldom considered in previous papers. Thirdly, endogenous direct channel delivery time is also studied in the dual-channel supply chain selling deteriorating products. The endogenous direct channel delivery time is studied by Hua et al. (2010) and Xu et al. (2012). However, how product deterioration affects the pricing, inventory and delivery time decisions are unclear in their research. So, based on their work, the endogenous delivery lead time problem is studied in a dual channel supply chain under product deterioration. Lastly, it is found that a revenue sharing and two part tariff contract can perfectly coordinate the dual channel supply chain with proper coordinating strategies.

3 Model formulation and solution analysis

The dual-channel supply chain consists of a vendor and a retailer. The supply chain structure is presented in **Fig. 1(a)**, in which the vendor distributes the products to customers through both the online channel and the retail channel (Chiang et al., 2003; Xu et al., 2012). As introduced above, the two parties' inventory systems are also considered, which are shown in **Fig.1(b)**. The mathematical models of the problems are formulated and algorithms are proposed to solve the models in the following subsections.



(a) Supply chain structure (b) Inventory system

Fig. 1 (a) Supply chain structure; (b) Inventory level with respect to time for the vendor and the retailer

3.1 Notation

Notation in this paper are presented in **Tab.2**.

3.2 Model formulation

Assuming that the vendor is the leader of the supply chain. Firstly, the vendor procures a large quantity of deteriorating products, noted by Q_v. Then the vendor sells the products to the downstream retailer, as well as to the end customers through direct channel. Under receiving the ordered products, the retailer sells the products to the customers. Customers can purchase the products either from the direct channel or the retail channel. As shown in **Fig.1(b)**, the vendor's inventory level depletes due to three reasons: the direct channel demand, the retail channel orders and deteriorated quantities. The retailer's inventory depletes due to retail channel demand and product deterioration.

Firstly, a centralized model is studied , i.e., the vendor and the retailer are vertically integrated. There are four decision variables: the retail price p_r, the direct channel price p_v, the vendor's inventory scale parameter n and the retailer's ordering cycle length T. Then, a decentralized model is studied, in which the vendor acts as the Stackelberg leader and the retailer as the follower. In this gaming problem, both of the vendor and the retailer make their own decisions to maximize their individual profit. The gaming sequence is: (1) The vendor announces the wholesale price w, the direct sale price p_v and the scale parameter n. (2) The retailer sets selling price p_r and ordering cycle length T under knowing the vendor's announced decisions.

Following the studies of Yue & Liu (2006), Huang & Swaminathan (2009), Hua et al. (2010) and Chen et al. (2012), demand functions of the two channels are linear in self and cross price affects with the same sensitive parameters. In addition to quantity deterioration, quality deterioration is also considered in this paper. Following the study of Wang & Li (2012), Fibich et al. (2003), Kopalle et al. (1996) and Sorger (1988), we assume that the product quality is exponentially decreasing with time with a rate of μ. Also, it is

Tab. 3 Notation

Decision variables	
p_v	price of the direct channel, \$/unit
p_r	price of the retail channel, \$/unit
T	retailer's replenishment cycle time, day
n	multiple of retailer's cycle time, an integer number.
w	wholesale price of the vendor, \$/unit
β	sharing rate of retailer's revenue to the vendor.
F	a lump sum fee transferred from the vendor to the retailer, \$
L	delivery lead time in the direct channel, day.
Constant parameters	
θ	deterioration rate of vendor and retailer.
μ	quality dropping rate.
a	total potential market size.
α	direct channel market share. The retail channel market share is $1-\alpha$.
b	coefficient of the price elasticity of demand rate.
r	degree of product substitution of the two channels.
c_v	vendor's purchasing cost per unit item, \$/unit
h_v, h_r	vendor's and retailer's holding cost per unit item per unit time, respectively, \$/unit/day
A_v, A_r	vendor's and retailer's fixed cost per order, respectively, including order processing cost, transportation cost, warehouse operating cost, etc, \$/time
γ	vendor's bargain power. Retailer's bargain power is $1-\gamma$, $0 \leq \gamma \leq 1$.
c_L	investment to reduce delivery time, which is formulated as $c_L = \frac{s_3}{L+s_4}$. s_3 and s_4 are constant parameters, \$/time
Dependent variables	
D_v, D_r	vendor's and retailer's demand rate, respectively.
Q_v	vendor's ordering quantity.
Q_{rj}	retailer's ordering quantity in phase $j(j=1,2,...,n)$.
T_v	vendor's replenishment cycle time, $T_v = nT$.
$I_{vj}(t), I_{rj}(t)$	vendor's and retailer's inventory level with respect to time in the jth phase, respectively, where $j=1,2,...,n$.
SR_v, SR_r	vendor's and retailer's total sales revenue, respectively.
WR_v	vendor's wholesale total revenue.
HC_v, HC_r	vendor's and retailer's total inventory holding cost, respectively.
PC_v, PC_r	vendor's and retailer's total purchasing cost, respectively.
OC_v, OC_r	vendor's and retailer's ordering cost, respectively.
HQ_v, HQ_r	vendor's and retailer's accumulated holding quantity in a single cycle, respectively.
TP_v, TP_r, TP_{sc}	vendor's, retailer's and supply chain's per unit time profit, respectively.
$\Phi_r, \Phi_v, \Phi_{sc}$	percentage profit increase for retailer ,vendor and supply chain.
$\overline{q}^c, \overline{q}^d$	total average ordered quantity for centralized and decentralized model, respectively.
$\overline{D}^c, \overline{D}^d$	total average demand for centralized and decentralized model, respectively.
$B(F)$	Nash bargain function.

realistic that retail channel buyers can touch the products and feel the quality changes (especially for fresh vegetables, meats and fruits). So, the retail channel demand is linked to the products' real time quality. Base on the assumptions, the demand functions in both channels can be formulated as:

$$D_v(p_v, p_r) = \alpha a - bp_v + rp_r, \tag{1}$$

$$D_r(p_v, p_r) = [(1-\alpha)a - bp_r + rp_v]e^{-\mu t}, t \in [0, nT]. \tag{2}$$

In the following analysis, for notational convenience, demand rate of the vendor's direct channel is marked by D_v, and that of the retail channel is marked by D_r. Also, the initial demand rate of the retail channel (i.e., $(1-\alpha)a - bp_r + rp_v$) is noted as d_r. The demand rates of the two channels are linked to the selling prices in both channels. The total potential market size is a. α $(0 < \alpha < 1\ )$ denotes the customer's preference for the direct channel, and a higher α means more customers will choose the direct channel. Parameter b is the coefficient of the price elasticity of demand rate. Parameter r indicates the degree of the substitution of the products sold via the two channels. To make sure own price effect is greater than cross price effect, the condition $b > r$ should be satisfied. The parameter μ denotes the quality dropping rate. For a higher μ, the product quality drops more fast with time. Specially, when $t = 0$, the product quality is 1, which means the product is totally fresh.

To concentrate on the research targets and to ease the analysis, other assumptions should be made.

(1) In the base model, the vendor's direct channel delivery time is assumed to be zero. This assumption is relaxed in the model extension by assuming the delivery time is endogenous and it will affect demands in both channels (Hua et al., 2010).

(2) Shortages in both channels are not allowed.

(3) Time horizon is infinite for the vendor and the retailer.

(4) The market size and deterioration rate do not change with time.

(5) No cost is incurred to deal with the deteriorated products. When products are deteriorated, firms will throw them away without any cost.

3.3 Retailer's profit

In this subsection, the retailer's profit is calculated. According to previous studies on deteriorating inventory problems, based on the inventory system depicted in **Fig.1(b)**, some calculations about the inventory level, the ordering quantities, and the total inventory holding quantities for the retailer can be made.

Following Ghare & Schrader (1963), the retailer's inventory level in phase j satisfies the differential equation

$$\dot{I}_{rj}(t) = -\theta I_{rj}(t) - d_r e^{-\mu t}, t \in [(j-1)T, jT], j = 1, 2, ..., n. \tag{3}$$

with boundary conditions $I_{rj}(t = jT) = 0$, $I_{rj}[t = (j-1)T] = Q_{rj}$. The inventory level with respect to time t can be derived by solving differential equation (3) as

$$I_{rj}(t) = \frac{d_r}{\theta - \mu}(e^{(\theta-\mu)jT} - e^{(\theta-\mu)t})e^{-\theta t}, t \in [(j-1)T, jT], j = 1, 2, ..., n. \tag{4}$$

The ordering quantities of the retailer in phase j can be obtained by equating t to $(j-1)T$, that is

$$Q_{rj} = I_{rj}[t = (j-1)T] = \frac{d_r}{\theta - \mu}(e^{(\theta-\mu)T} - 1)e^{-(j-1)\mu T}, j = 1, 2, ..., n. \tag{5}$$

The retailer's total inventory holding quantities in phase j is

$$HQ_{rj} = \int_{(j-1)T}^{jT} I_{rj}(t)\mathrm{d}t = \frac{d_r}{\theta - \mu}\left(\frac{e^{\theta T} - 1}{\theta} - \frac{e^{\mu T} - 1}{\mu}\right)e^{-\mu jT}, j = 1, 2, ..., n. \tag{6}$$

After obtaining the inventory level, the ordering quantities, and the total inventory holding quantities, the related revenue and the costs of the retailer can be obtained as follows.

(1) Retailer's total sales revenue (SR_r)
The retailer's total revenue comes from the sales of the deteriorating products. The total sales revenue in a cycle ($t \in [0, nT]$) can be calculated as

$$SR_r = p_r \int_0^{nT} D_r \mathrm{d}t = p_r d_r \frac{1 - e^{-\mu nT}}{\mu}. \tag{7}$$

(2) Retailer's total holding cost (HC_r)
After receiving the ordered products from the vendor, the retailer stores its products in the inventory. The retailer needs to pay for the holding cost for its inventory. The total inventory holding cost in an ordering cycle can be calculated as

$$HC_r = h_r \sum_{j=1}^{n} HQ_{rj} = h_r \frac{d_r}{\theta - \mu} \left(\frac{e^{\theta T} - 1}{\theta} - \frac{e^{\mu T} - 1}{\mu} \right) \frac{1 - e^{-\mu nT}}{e^{\mu T} - 1}. \tag{8}$$

(3)Retailer's total purchasing cost (PC_r)
The retailer purchases its products from the vendor with the wholesale price w. Thus, the total purchasing cost for the retailer in an ordering cycle can be calculated as

$$PC_r = w \sum_{j=1}^{n} Q_{rj} = w \frac{d_r}{\theta - \mu} (e^{(\theta-\mu)T} - 1) \frac{1 - e^{-\mu nT}}{1 - e^{-\mu T}}. \tag{9}$$

(4) Retailer's total ordering cost (OC_r)
When ordering from the vendor, the retailer need to pay a lump sum fee A_r per time. In the cycle $t \in [0, nT]$, it will order n times. So the total ordering cost in n phases is

$$OC_r = nA_r. \tag{10}$$

Based on the revenue and cost functions described above, the total profit per unit time for the retailer can be obtained

$$\begin{aligned} TP_r(p_r, T) &= \frac{1}{nT}[SR_r - PC_r - HC_r - OC_r] \\ &= \frac{1}{nT}\{p_r d_r \frac{1 - e^{-\mu nT}}{\mu} \\ &-h_r \frac{d_r}{\theta - \mu} \left(\frac{e^{\theta T} - 1}{\theta} - \frac{e^{\mu T} - 1}{\mu} \right) \frac{1 - e^{-\mu nT}}{e^{\mu T} - 1} \\ &-w \frac{d_r}{\theta - \mu} (e^{(\theta-\mu)T} - 1) \frac{1 - e^{-\mu nT}}{1 - e^{-\mu T}} \\ &-nA_r\}. \end{aligned} \tag{11}$$

The first part of retailer's profit is the sales revenue. The second part is the retailer's total inventory cost. The fourth part is the total purchasing cost. The last part is the fixed ordering cost.

3.4 Vendor's profit

In this subsection, the vendor's profit is calculated. In practice, vendors usually have inventory holding cost advantage over retailers. So, for the vendor, the replenishment cycle (T_v) is much longer than that of the retailer, which is n times as much as the retailer's ordering cycle (i.e., $T_v = nT$). It is more complex for the calculation of the vendor's ordering quantity and total inventory holding quantity in time interval $t \in [0, nT]$. There are n phases in the vendor's inventory system. For every phase j,($j = 1, 2, ..., n$), the vendor's inventory level satisfies the following differential equation.

$$\dot{I}_{vj}(t) = -\theta I_{vj}(t) - D_v, t \in [(j-1)T, jT], j = 1, 2, ..., n. \tag{12}$$

with boundary conditions (1) $I_{v(j-1)}[t = (j-1)T] - I_{vj}[t = (j-1)T] = Q_{rj}$ for $j = 2, 3, ..., n$, (2) $I_{vn}(t = nT) = 0$ and (3) $I_{v1}(t = 0) = Q_v - Q_{r1}$.

Lemma 1 *The inventory level for the vendor in a cycle in phase j is*

$$I_{vj}(t) = \frac{D_v}{\theta}(e^{\theta(nT-t)} - 1) + \frac{d_r}{\theta - \mu}(e^{(\theta-\mu)nT} - e^{(\theta-\mu)jT})e^{-\theta t}, t \in [(j-1)T, jT] \tag{13}$$

in which, $j = 1, 2, ..., n$.

After obtaining the vendor's inventory level, the total inventory holding quantity in phase j can be calculated as

$$HQ_{vj} = \int_{(j-1)T}^{jT} I_{vj}(t)\mathrm{d}t = \frac{D_v e^{\theta nT}(e^{\theta T} - 1)}{\theta^2}e^{-j\theta T} - \frac{D_v T}{\theta} + \frac{d_r(e^{\theta T} - 1)}{(\theta - \mu)\theta}(e^{(\theta-\mu)nT}e^{-j\theta T} - e^{-j\mu T}). \tag{14}$$

The total inventory holding quantity in time interval $t \in [0, nT]$ can be calculated as

$$HQ_v = \sum_{j=1}^{n} HQ_{vj} = D_v\frac{e^{\theta nT} - \theta nT - 1}{\theta^2} + \frac{d_r}{(\theta - \mu)\theta}\left(e^{(\theta-\mu)nT}(1 - e^{-\theta nT}) + (e^{\theta T} - 1)\frac{1 - e^{-\mu nT}}{1 - e^{\mu T}}\right). \tag{15}$$

As shown in Fig. 1(b), at time zero, the vendor receives quantity Q_v and then transport Q_{r1} to the retailer. Then, the rest of the quantity is stocked in the vendor's warehouse. According to equation (13), the initial inventory level can be calculated as $I_{v1}(t = 0)$. So, the vendor's ordering quantity can be obtained as

$$Q_v = I_{v1}(t = 0) + Q_{r1} = \frac{D_v}{\theta}(e^{\theta nT} - 1) + \frac{d_r}{\theta - \mu}(e^{(\theta-\mu)nT} - 1). \tag{16}$$

After obtaining the inventory level, the ordering quantities, and the total inventory holding quantities, the revenue and cost of the vendor can be expressed as follows.

(1) Vendor's total sales revenue (SR_v)

The vendor's revenue comes from both channels. It gains revenue by selling to customers through direct channel. The total revenue in a cycle can be calculated as

$$SR_v = p_v D_v nT. \tag{17}$$

(2) Vendor's total wholesale revenue (WR_v)

The vendor also gains revenue by wholesaling products to the downstream retailer, which can be expressed as

$$WR_v = w\sum_{j=1}^{n} Q_{rj} = w\frac{d_r}{\theta-\mu}(e^{(\theta-\mu)T}-1)\frac{1-e^{-\mu nT}}{1-e^{-\mu T}}. \tag{18}$$

(3) Vendor's total inventory holding cost (HC_v)

During the selling period, the unsold products are stored in the warehouse. The vendor needs to pay for holding the inventory. The total inventory holding cost in a selling cycle can be calculated as

$$HC_v = h_v HQ_v = h_v D_v \frac{e^{\theta nT}-\theta nT-1}{\theta^2} + h_v\frac{d_r}{(\theta-\mu)\theta}\left(e^{(\theta-\mu)nT}(1-e^{-\theta nT})+(e^{\theta T}-1)\frac{1-e^{-\mu nT}}{1-e^{\mu T}}\right). \tag{19}$$

(4) Vendor's total purchasing cost (PC_v)

Before selling the products, the vendor needs to replenish a large quantity of products from its upstream suppliers, such as some large farms or food producers, which a unit replenishment cost of c_v. Thus, the total purchasing cost in a cycle can be calculated as

$$PC_v = c_v Q_v = c_v\frac{D_v}{\theta}(e^{\theta nT}-1) + c_v\frac{d_r}{\theta-\mu}(e^{(\theta-\mu)nT}-1). \tag{20}$$

(5) Vendor's ordering cost (OC_v)

When ordering from the suppliers, the vendor need to pay a lump sum fee, which is

$$OC_v = A_v. \tag{21}$$

Based on the elements described above, the total profit per unit time for the vendor is

$$\begin{aligned}
TP_v(p_v, w, n) &= \frac{1}{nT}[SR_v + WR_v - HC_v - PC_v - OC_v]\\
&= \frac{1}{nT}\{p_v D_v nT\\
&+w\frac{d_r}{\theta-\mu}(e^{(\theta-\mu)T}-1)\frac{1-e^{-\mu nT}}{1-e^{-\mu T}}\\
&-h_v D_v\frac{e^{\theta nT}-\theta nT-1}{\theta^2} - h_v\frac{d_r}{(\theta-\mu)\theta}\left(e^{(\theta-\mu)nT}(1-e^{-\theta nT})+(e^{\theta T}-1)\frac{1-e^{-\mu nT}}{1-e^{\mu T}}\right)\\
&-c_v\frac{D_v}{\theta}(e^{\theta nT}-1) - c_v\frac{d_r}{\theta-\mu}(e^{(\theta-\mu)nT}-1)\\
&-A_v\}.
\end{aligned} \tag{22}$$

The first part of vendor's profit is the sales revenue from the direct online channels. The second part is the wholesale revenue from the retail channel. The third part is the vendor's total inventory cost in an ordering cycle. The fourth part is the total purchasing cost for the vendor in a cycle. The last part is the fixed cost in a cycle.

3.5 Analysis of the centralized problem

In this subsection, a centralized dual-channel supply chain is considered, in which the vendor and the retailer are vertically integrated. In this case, the wholesale price is only used to divide the total profit between the vendor and the retailer. The decision variables are p_v, p_r, n and T. The unit time total profit function of the supply chain is

$$
\begin{aligned}
TP_{sc}(p_v, p_r, n, T) &= TP_v + TP_r \\
&= \frac{1}{nT}\{p_v D_v nT + p_r d_r \frac{1-e^{-\mu nT}}{\mu} \\
&-h_v D_v \frac{e^{\theta nT} - \theta nT - 1}{\theta^2} - h_v \frac{d_r}{(\theta-\mu)\theta}\left(e^{(\theta-\mu)nT}(1-e^{-\theta nT}) + (e^{\theta T}-1)\frac{1-e^{-\mu nT}}{1-e^{\mu T}}\right) \\
&-h_r \frac{d_r}{\theta-\mu}\left(\frac{e^{\theta T}-1}{\theta} - \frac{e^{\mu T}-1}{\mu}\right)\frac{1-e^{-\mu nT}}{e^{\mu T}-1} \\
&-c_v \frac{D_v}{\theta}(e^{\theta nT}-1) - c_v \frac{d_r}{\theta-\mu}(e^{(\theta-\mu)nT}-1) \\
&-nA_r - A_v\}.
\end{aligned}
\tag{23}
$$

The problem is to maximize the above function by finding optimal values of p_v, p_r, n and T. Through analysis, the following proposition can be obtained.

Proposition 1 *When θ and μ are relatively small,*

(1) For constant n and T, TP_{sc} is jointly concave in p_v and p_r.The optimal price in the direct and retail channel can be respectively expressed as

$$p_v^{c*} \approx \frac{B_2 r + 2bB_1X_1 + rX_1B_2 - arX_1 - arX_1^2 + \alpha arX_1^2 - 2\alpha abX_1 + \alpha arX_1}{r^2X_1^2 - 4b^2X_1 + 2r^2X_1 + r^2}, \tag{24}$$

$$p_r^{c*} \approx \frac{2bB_2 + rB_1 + rB_1X_1 - 2abX_1 - \alpha ar + 2\alpha abX_1 - \alpha arX_1}{r^2X_1^2 - 4b^2X_1 + 2r^2X_1 + r^2}. \tag{25}$$

(2) For constant p_v, p_r and n, TP_{sc} is concave in T.

$$T^{c*} \approx \sqrt{\frac{2(nA_r + A_v)}{p_r d_r \mu n^2 + h_v[D_v n^2 + d_r(n-1)n] + h_r d_r n - c_v[D_v\theta n^2 + dr(\theta-\mu)n^2]}} \tag{26}$$

(3) For constant p_v, p_r and T, TP_{sc} is concave in n.

$$n^{c*} \approx \sqrt{\frac{2A_v}{p_r d_r \mu T^2 + h_v[D_v T^2 + d_r T^2] - c_v[D_v\theta T^2 + dr(\theta-\mu)T^2]}} \tag{27}$$

$X_1 = 1 - \frac{\mu nT}{2}$,
$B_1 = -h_v b\frac{nT}{2} + h_v r\frac{(n-1)T}{2} + h_r r\frac{T}{2} - c_v b(1-\frac{\theta nT}{2}) + c_v r(1-\frac{(\theta-\mu)nT}{2})$,
$B_2 = h_v r\frac{nT}{2} - h_v b\frac{(n-1)T}{2} - h_r b\frac{T}{2} + c_v r(1-\frac{\theta nT}{2}) - c_v b(1-\frac{(\theta-\mu)nT}{2})$.

Proposition 1 indicates that TP_{sc} can be maximized for constant n and T. By updating the values of n and T, the optimal decisions can be found. So, a multi-stages searching method is designed to determine the optimal solutions for the model. The algorithm is presented in **Tab.4**.

3.6 Analysis of the decentralized problem

In this section, a decentralized supply chain is studied, in which both the vendor and the retailer make their own decisions to maximize their individual profit. Firstly, the vendor, as the Stackelberg gaming leader, determines its wholesale price w, the direct channel price p_v and scale parameter n. Then, the retailer, as the

Tab. 4 Algorithm to solve the centralized problem

Algorithm 1	
Step 1:	Input parameters α, a, b, r, h_v, h_r, c_v, A_v, A_r, θ, μ. Set $n = 1$;
Step 2:	Set $k = 1$. Initialize the value of $T^{(k)} = \Delta T \cdot k$ in which $\Delta T = 10^{-2}$ is the searching step size;
Step 3:	Substitute $T^{(k)}$ and n into equations (24) and (25), and obtain the corresponding values of $p_v^{(k)}$, $p_r^{(k)}$, $TP_{sc}\left(p_v^{(k)}, p_r^{(k)}, n, T^{(k)}\right)$;
Step 4:	Set $k = k + 1$, then $T^{(k)} = \Delta T \cdot (k + 1)$. Go back to Step 3 and obtain the values of $p_v^{(k+1)}$, $p_r^{(k+1)}$,$TP_{sc}(n, T^{(k+1)}, p_v^{(k+1)}, p_r^{(k+1)})$;
Step 5:	Repeat Step 4, stop until k satisfies conditions $$TP_{sc}\left(n, T^{(k)}, p_v^{(k)}, p_r^{(k)}\right) \geqslant TP_{sc}\left(n, T^{(k-1)}, p_v^{(k-1)}, p_r^{(k-1)}\right),$$ $$TP_{sc}\left(n, T^{(k)}, p_v^{(k)}, p_r^{(k)}\right) \geqslant TP_{sc}\left(n, T^{(k+1)}, p_v^{(k+1)}, p_r^{(k+1)}\right),$$ note $k^*_{(n)} = k$, $T^*_{(n)} = T^{(k)}$, $p^*_{v(n)} = p_v^{(k)}$, $p^*_{r(n)} = p_r^{(k)}$ and $TP^*_{sc(n)} = TP_{sc}(n, T^{(k)}, p_v^{(k)}, p_r^{(k)})$.
Step 6:	Set $n = n + 1$, repeat step 2-5 and obtain the corresponding values of $k^*_{(n+1)}$, $T^*_{(n+1)}$, $p^*_{v(n+1)}$, $p^*_{r(n+1)}$, $TP^*_{sc(n+1)}$.
Step 7:	Repeat Step 6 until n satisfies $$TP^*_{sc}\left(n, p^{(*)}_{v(n)}, p^{(*)}_{r(n)}, T^*_{(n)}\right) \geqslant TP^*_{sc}\left(n+1, p^{(*)}_{v(n+1)}, p^{(*)}_{r(n+1)}, T^*_{(n+1)}\right),$$ $$TP^*_{sc}\left(n, p^{(*)}_{v(n)}, p^{(*)}_{r(n)}, T^*_{(n)}\right) \geqslant TP^*_{sc}\left(n-1, p^{(*)}_{v(n-1)}, p^{(*)}_{r(n-1)}, T^*_{(n-1)}\right).$$
	Output $n^{c*} = n$, $T^{c*} = T^*_{(n^{c*})}$,$p_v^{c*} = p^*_{v(n^{c*})}$, $p_r^{c*} = p^*_{r(n^{c*})}$, $TP^{c*}_{sc} = TP^*_{sc(n^{c*})}$.

follower, sets its sales price p_r and replenishment cycle length T based on the vendor's decisions. Properties of the model are listed in proposition 2 and an algorithm is designed to find the optimal equilibriums.

Proposition 2 *When θ and μ are relatively small,*

(1) For constant T and n, the optimal prices p_v^{d*}, w^{d*} and p_r^{d*} can be expressed respectively as

$$p_v^{d*} \approx \frac{2bC_2 + 2rC_1 - 2\alpha ab - arX_7 + \alpha ar + \alpha arX_7}{2(r^2X_7 - 2b^2 + r^2)}, \tag{28}$$

$$w^{d*} \approx \frac{2b^2C_1 - r^2C_1 - ab^2X_7 + \alpha ab^2X_7 + brX_7C_2 - \alpha abrX_7}{X_7b(r^2X_7 - 2b^2 + r^2)}, \tag{29}$$

$$p_r^{d*} \approx \frac{1}{2b}((1-\alpha)a + rp_v^{d*} + bh_rX_4 + bX_7w^{d*}). \tag{30}$$

(2) For constant price parameters p_v, p_r, w and n, retailer's profit function is concave in T.

$$T^{d*} \approx \sqrt{\frac{2A_r}{p_rd_r\mu n + h_rd_r + wd_r(\theta-\mu)n}} \tag{31}$$

(3) For constant price parameters p_v, p_r, w and T, vendor's profit function is concave in n.

$$n^{d*} \approx \sqrt{\frac{2A_v}{wd_r(\theta-\mu)T^2 + h_v[D_vT^2 + d_rT^2] - c_v[D_v\theta T^2 + dr(\theta-\mu)T^2]}} \tag{32}$$

Tab. 5 Algorithm to solve the decentralized problem

Algorithm 2	
Step 1:	Input parameters α, a, b, r, h_v, h_r, c_v, A_v, A_r, θ, μ. Set $n=1$;
Step 2:	Set $k=1$. Initialize the value of $T^{(k)}=\Delta T\cdot k$, in which $\Delta T=10^{-2}$ is the searching step size;
Step 3:	Substitute $T^{(k)}$ and n into equations (28)-(30) and obtain the corresponding values of $p_v^{(k)}$, $w^{(k)}$,$p_r^{(k)}$,$TP_v\left(n,p_v^{(k)},w^{(k)}\right)$, and $TP_r\left(T^{(k)},p_r^{(k)}\right)$;
Step 4:	Set $k=k+1$, then $T^{(k)}=\Delta T\cdot(k+1)$. Go back to Step 3 and obtain the values of $p_v^{(k+1)}$, $w_v^{(k+1)}$, $p_r^{(k+1)}$, $TP_v\left(n,p_v^{(k+1)},w^{(k+1)}\right)$ and $TP_r\left(T^{(k+1)},p_r^{(k+1)}\right)$;
Step 5:	Repeat Step 4, stop until k satisfies conditions $$TP_r\left(T^{(k)},p_r^{(k)}\right)\geqslant TP_r\left(T^{(k-1)},p_r^{(k-1)}\right),$$ $$TP_r\left(T^{(k)},p_r^{(k)}\right)\geqslant TP_r\left(T^{(k+1)},p_r^{(k+1)}\right),$$ note $k^*_{(n)}=k$, $T^*_{(n)}=T^{(k)}$, $p^*_{v(n)}=p_v^{(k)}$, $p^*_{r(n)}=p_r^{(k)}$, $w^*_{(n)}=w^{(k)}$, $TP^*_v(n,p_v^{(k)},w^{(k)})$ and $TP^*_{r(n)}(T^{(k)},p_r^{(k)})$.
Step 6:	Set $n=n+1$, repeat step 2-5 and obtain the corresponding values of $k^*_{(n+1)}$, $T^*_{(n+1)}$, $p^*_{v(n+1)}$, $p^*_{r(n+1)}$, $w^*_{(n+1)}$, $TP^*_v\left(n+1,p^{(*)}_{v(n+1)},w^{(*)}_{(n+1)}\right)$ and $TP^*_{r(n+1)}\left(T^{(*)}_{(n+1)},p^{(*)}_{r(n+1)}\right)$.
Step 7:	Repeat Step 6 until n satisfies $$TP^*_v\left(n,p^{(*)}_{v(n)},w^{(*)}_{(n)}\right)\geqslant TP^*_v\left(n+1,p^{(*)}_{v(n+1)},w^{(*)}_{(n+1)}\right),$$ $$TP^*_v\left(n,p^{(*)}_{v(n)},w^{(*)}_{(n)}\right)\geqslant TP^*_v\left(n-1,p^{(*)}_{v(n-1)},w^{(*)}_{(n-1)}\right).$$ Output $n^{d*}=n$, $T^{d*}=T^*_{(n^{d*})}$,$p_v^{d*}=p^*_{v(n^{d*})}$, $p_r^{d*}=p^*_{r(n^{d*})}$, $w^{d*}=w^*_{(n^{d*})}$, $TP_v^{d*}=TP^*_{v(n^{d*})}$, $TP_r^{d*}=TP^*_{r(n^{d*})}$.

$$X_7=1-\frac{(\theta-\mu)nT}{2}n,$$
$$C_1=\left\{\frac{h_rbT}{4}+\frac{h_vrnT}{4}-\frac{h_vb(n-1)T}{4}+\frac{c_vr(1-\frac{\theta nT}{2})}{2}-\frac{c_vb(1-\frac{(\theta-\mu)nT}{2})}{2}\right\}(1-\frac{(\theta-\mu)nT}{2})n,$$
$$C_2=h_v\frac{nT}{2}(\frac{r^2}{2b}-b)+\frac{h_vr(n-1)T}{4}+c_v(1-\frac{\theta nT}{2})(\frac{r^2}{2b}-b)+\frac{c_v(1-\frac{(\theta-\mu)nT}{2})r}{2}-\frac{h_rrT}{4}$$

Proposition 2 indicates that the equilibrium for the decentralized problem exists. However, due to the complexity, the explicit solutions for parameters n and T can not be derive. Instead, another algorithm is designed to solve the problem, which is presented in **Tab.5**.

4 Numerical tests

In this section, the proposed models are exemplified by numerical examples, in which the initial values of the parameters are set as follows: $\alpha=0.5$, $a=500$, $b=20$, $r=5$, $h_v=0.05$\$/unit/time, $h_r=0.2$\$/unit/time, $c_v=4$\$/unit, $A_v=8000$\$, $A_r=100$\$, $\theta=0.01$, $\mu=0.01$. Superscripts $(\cdot)^{c*}$ and $(\cdot)^{d*}$ are used to denote optimal decisions under centralized and decentralized problems respectively in the following analysis.

4.1 Examples for centralized and decentralized models

For the centralized model, using Algorithm 1, the optimal decisions can be determined: $p_v^{c*}=10.99$\$/unit, $p_r^{c*}=11.22$\$/unit, $T^{c*}=2.92$day, $n^{c*}=10$. The maximum profit is $TP_{sc}^{c*}=562.34$\$.

Tab. 6 Optimal centralized and decentralized decisions for different θ

		Centralized Supply Chain					Decentralized supply chain							
		p_v^{c*}	p_r^{c*}	T^{c*}	n^{c*}	TP^{c*}	p_v^{d*}	w^{d*}	p_r^{d*}	T^{d*}	n^{d*}	TP_v^{d*}	TP_r^{d*}	TP^{d*}
	0	10.72	11.04	3.50	11	690.25	10.83	10.82	13.16	2.94	15	581.05	33.87	614.92
	0.005	10.86	11.14	3.30	10	624.77	10.98	10.85	13.23	2.85	13	516.93	32.82	549.75
θ	0.010	10.99	11.22	2.92	10	562.34	11.11	10.89	13.29	2.69	12	458.46	30.15	488.62
	0.015	11.09	11.30	2.90	9	505.95	11.21	10.91	13.34	2.59	11	404.53	28.13	432.66
	0.020	11.19	11.38	2.65	9	450.68	11.34	10.98	13.42	2.41	11	353.82	23.27	377.09

For the decentralized model, Algorithm 2 is utilized to search for the optimal decisions. For the vendor, the optimal decision is $p_v^{d*} = 11.11$\$/unit, $w^{d*} = 10.89$\$/unit, $n^{d*} = 12$, maximum profit is $TP_v^{d*} = 458.46$\$. For the retailer, the optimal decision is $p_r^{d*} = 13.29$\$/unit, $T^{d*} = 2.69$day, maximum profit is $TP_r^{d*} = 30.15$\$. The total profit of the supply chain is $TP_{sc}^{d*} = 488.62$\$, which is less than that of the centralized supply chain.

In the centralized model, the *Total Waste Rate* (defined as $\left(1 - \frac{Total\ Demand}{Total\ Ordered\ Quantity}\right) \times 100\%$) is 13.61%. However, in the decentralized model, the *Total Waste Rate* rises to 15.09%. So, it is worth noting that supply chain integration not only results in higher profit, but also helps to reduce the wastes due to product deterioration.

4.2 Sensitivity analysis on equilibrium strategies

In this subsection, sensitivity analysis is carried out on the equilibrium strategies with respect to key system parameters θ, μ, α, r, h_v,h_r,A_v,A_r and c_v by varying one parameter once and keeping other parameters fixed.

(1)Sensitivity analysis of deterioration rate θ

Tab. 6 shows that, in both the centralized and the decentralized models, for a higher deterioration rate, the direct channel price and the retail channel price increases. This is because a higher deterioration rate means more products are wasted, thus leads to higher deterioration cost, especially for the vendor with larger inventory holding quantities. To protect its profit margin, the firm has strong incentives to enhance both channels' prices. Besides, when deterioration rate is higher, the vendor and the retailer suffers more from product deterioration. To reduce the waste rate and cut down the deterioration cost, the replenishment cycles of the vendor and the retailer are compressed. In summary, in the two models, for the same value of deterioration rate, the optimal price for the direct channel under decentralization is slightly higher than that of the centralized model. Due to the double marginalization effect, the retail channel price in the decentralized case is much higher than that of the centralized model.

Then, the average quantity and demand change with respect to deterioration rate is shown in **Fig. 2(a)**. In both cases, the average purchased quantity is increasing in deterioration rate. However, on the contrary, the increased average quantity does not result in the improvement of market demand. As it is shown, demand rate decreases dramatically with deterioration rate. The gap between the average quantity and the demand rate is expending with higher deterioration rate, which means the wasted quantity of the products is increasing in deterioration rate. Although the size of the gaps for the decentralized and the centralized cases are of similar sizes, the total waste rate in the decentralized case is higher than that of the centralized model.

(2) Sensitivity analysis of quality losing rate μ

Tab. 7 Optimal centralized and decentralized decisions for different μ

		Centralized Supply Chain					Decentralized supply chain							
		p_v^{c*}	p_r^{c*}	T^{c*}	n^{c*}	TP^{c*}	p_v^{d*}	w^{d*}	p_r^{d*}	T^{d*}	n^{d*}	TP_v^{d*}	TP_r^{d*}	TP^{d*}
μ	0	11.09	11.21	3.04	10	617.12	11.18	10.82	13.30	3.04	11	485.80	47.16	532.96
	0.005	11.03	11.21	2.97	10	588.39	11.11	10.82	13.29	2.89	11	471.41	39.68	511.08
	0.010	10.99	11.22	2.92	10	562.34	11.11	10.89	13.29	2.69	12	458.46	30.15	488.62
	0.015	10.94	11.24	3.17	9	540.89	11.07	10.91	13.28	2.63	12	446.46	24.05	470.51
	0.020	10.91	11.27	3.14	9	519.68	11.04	10.94	13.27	2.58	12	435.46	18.56	454.02

Tab. 8 Optimal centralized and decentralized decisions for different α and r

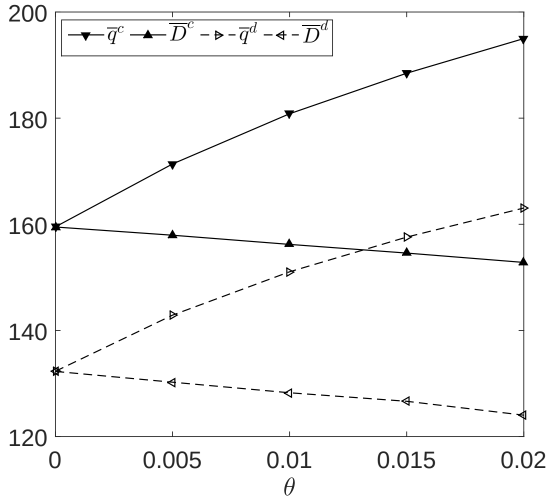
		Centralized Supply Chain					Decentralized supply chain							
		p_v^{c*}	p_r^{c*}	T^{c*}	n^{c*}	TP^{c*}	p_v^{d*}	w^{d*}	p_r^{d*}	T^{d*}	n^{d*}	TP_v^{d*}	TP_r^{d*}	TP^{d*}
α	0.450	10.46	11.69	2.85	10	557.67	10.60	11.42	14.09	2.30	14	407.37	47.82	455.19
	0.475	10.72	11.45	2.89	10	557.05	10.86	11.16	13.69	2.49	13	430.84	38.56	469.39
	0.500	10.99	11.22	2.92	10	562.34	11.11	10.89	13.29	2.69	12	458.46	30.15	488.62
	0.525	11.25	10.99	3.25	9	575.36	11.37	10.63	12.90	2.98	11	489.84	22.61	512.45
	0.550	11.51	10.76	3.28	9	592.53	11.62	10.35	12.50	3.26	10	525.66	15.93	541.60
r	1	11.04	11.16	2.91	10	561.83	11.13	10.96	13.77	2.30	14	416.36	45.77	462.14
	3	11.01	11.19	2.91	10	562.05	11.12	10.92	13.51	2.49	13	439.79	36.78	476.57
	5	10.99	11.22	2.92	10	562.34	11.11	10.89	13.29	2.69	12	458.46	30.15	488.62
	7	10.97	11.24	3.22	9	564.41	11.09	10.87	13.12	2.92	11	473.67	25.23	498.90
	9	10.96	11.26	3.22	9	564.77	11.06	10.85	12.95	3.11	10	486.53	21.58	508.11

Under the decentralized scenario, **Tab.7** shows that the direct channel price and retail channel price decreases, while the wholesale price increases in μ. From economic point of view, when the quality drops faster, the demand in the retail channel drops and the retailer will set a lower price to stimulate demand. For the vendor, to get a higher direct channel demand and to mitigate the double marginalization effect, it will set a lower direct channel price and a higher wholesale price. The drop of retail channel demand also leads to a lower inventory holding quantity, so that the two parties' ordering cycles are shortened. It is natural that higher quality losing rate leads to the drop of both parties' profit. Under the centralized scenario, as μ increases, p_v^{c*}, p_r^{c*}, $T_v^{d*}(T^{d*} \cdot n^{d*})$ and TP_{sc}^{c*} drops, which in line with the results in the decentralized scenario.

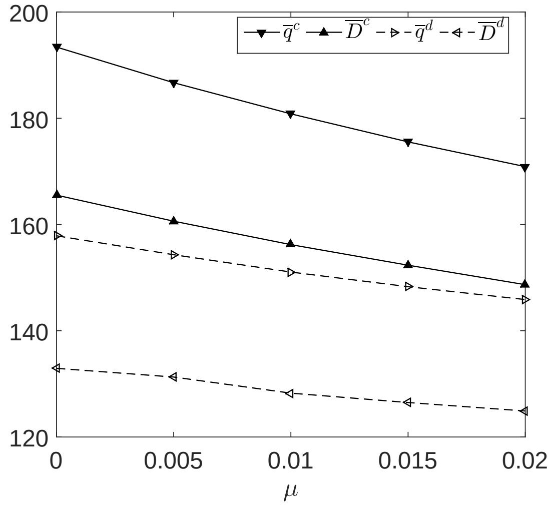
Then, the average quantity and demand change with respect to quality losing rate μ is shown in **Fig. 2(b)**. In both cases, the average purchased quantity and total average demand decline in μ. As it is shown, the average purchased quantity and total average demand drops in similar speed. The gap between the average quantity and the demand rate is shrinking with higher μ, which means the wasted quantity of the products is decreasing in μ. Although the size of the gaps for the decentralized and the centralized cases are of similar sizes, the total waste rate in the decentralized case is higher than that of the centralized model.

(3) Sensitivity analysis for market share α and competition intensity r

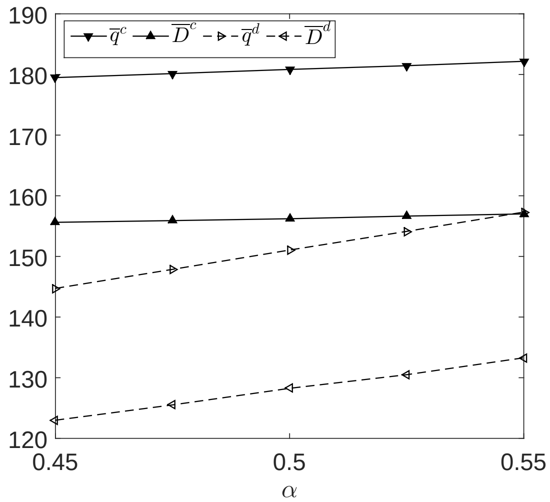
Tab. 8 shows that, when the direct channel market share (α) increases, for both the centralized and decentralized models, the direct channel price increases while the retail channel price decreases. The wholesale price drops with α in the decentralized case. The replenishment cycle for the retailer increases in α while vendor's ordering cycle decreases in both cases. For the decentralized model, when direct channel market share α increases, the vendor has more power in the market, so it can set a higher selling price. However, for the retailer, to survive on the market, it would set a lower selling price. In order to balance the revenue of the two channels, the vendor transfers some demand to the retailer by setting a lower wholesale price, so that the retailer can set a lower price and increase the retailer channel demand. The market share of direct channel contributes to the profit of the total supply chain profit under both cases and vendor's profit under decentralized case. However, the retailer's profit is hurt due to the drop of its market power. In **Fig. 2(c)**, it is depicted that the average quantity and demand are not sensitive to α in the centralized case. However, under the decentralized case, the average quantity and demand both increase with α, the gap



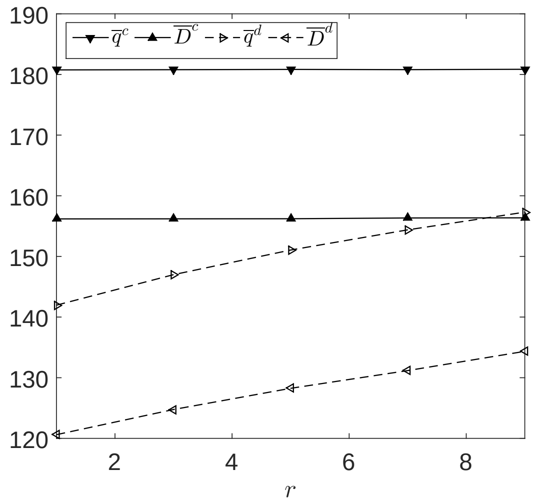
(a) Average quantity and demand change with θ



(b) Average quantity and demand change with μ



(c) Average quantity and demand change with α



(d) Average quantity and demand change with r

Fig. 2 Average quantity and demand change with respect to (a) θ (b) μ (c) α (d) r

remains unchanged, which means the direct channel power contributes to the improvement of demand and to the reduction of total waste.

Next, the influence of competition intensity to the optimal decisions are studied. Based on Chen et al. (2012), the competition intensity can be denoted as r, where $r = b - b^{'}$ ($b^{'}$is a constant). Note that, when $r = 0$, there is no competition between the two channels. In this paper, $b^{'}$ is set as $b^{'} = 15$, and the corresponding sets of r and b are $(r, b) = \{(1, 16), (3, 18), (5, 20), (7, 22), (9, 24)\}$. The result is presented in **Tab. 8**. It is shown that when the competition intensity increases, for both the centralized and decentralized models, the direct channel price decreases. The retail channel price increases in the centralized model, while decreases in the decentralized model. The ordering cycle of centralized model and the decentralized model are increasing in the competition intensity. It shows in **Fig. 2(d)** that the average ordering quantity and profit of centralized model are slightly sensitive to competition intensity. When competition intensity increases, an integrated vendor should increase the price gap between the direct and the retail channel to keep the demand, ordering quantity and profit stable. It also shows in **Fig. 2(d)** that both the average quantity and the demand increase with competition intensity in the decentralized case. For the decentralized model, the intensified competition will push the retailer to set a lower selling price. In response, to maintain the demand in both channels, the vendor will also set lower wholesale and direct selling price. This pricing strategy helps to mitigate the double marginalization effect between the two parties, thus results in an significant enhancement of total supply chain profit. However, when the vendor acts as the game leader, competition intensity only has positive effect on the vendors profit, while has negative effect on the retailers profit. In summary, in the centralized model, competition intensity has minor effects to the profitability of the supply chain. In the decentralized model, intense competition will benefit the vendor, while will hurt the retailer. Numerically, **Fig. 2(d)** also tells that the gap between average quantity and demand is not sensitive to competition intensity, while the waste rate drops under higher competition intensity.

(4) Sensitivity analysis for cost parameters h_v, h_r, A_v, A_r, c_v

We show the sensitivity results of h_v, h_r, A_v, A_r and c_v in **Tab. 9**. In the decentralized case, when the vendor's inventory holding cost h_v increases, p_v^{d*}, p_r^{d*} and w^{d*} increase, whereas TP_v^{d*}, TP_r^{d*} and TP_{sc}^{d*} decrease. A larger inventory holding cost h_v pushes the vendor to set higher selling and wholesale prices so as to protect its sales margin. The retailer will also increase its selling price under a higher wholesale price. The increase of prices results in the drop of market demand, thus leads to the drop of profit in both channels and the total supply chain. Although the dropped demand rate results in lower total inventory holding cost, it can not compensate the loss of demand decline. A similar rationale can be applied to the centralized case, in which prices for the two channels p_v^{c*} and p_r^{c*} increase and total supply chain profit TP_{sc}^{c*} decreases with a higher inventory holding cost h_v.

For a higher value of h_r, in the decentralized case, the vendor's wholesale price w^{d*} decrease, while the retailer's selling price p_r^{d*} increases and ordering cycle T^{d*} decreases. It is common that the retailer will enhance its market price and cut the ordering cycle to obtain higher sales margin and reduce its total inventory holding cost, although it leads to the drop of retailer's ordering quantity. Thus, to stimulate the retailer to order more products, the vendor sets a lower wholesale price. In the centralized case, higher inventory holding cost in the retail channel leads to the rise of both channels' prices.

For a higher value of parameter A_v, in the decentralized case, prices in the two channels (p_v^{d*}, p_r^{d*} and w^{d*}) and ordering cycle of the vendor $T_v^{d*}(T^{d*} \cdot n^{d*})$ increase, while the retailer's ordering cycle T^{d*} decreases. To reduce the high ordering cost, the vendor sets a longer ordering cycle. However, longer ordering cycle means larger ordering quantity and inventory holding cost. To balance the fixed ordering cost and total inventory holding cost, the vendor also sets higher prices to keep a low demand rate, thus it can achieve smaller ordering quantity, lower inventory level and lower inventory holding cost. No doubt that the increase of ordering cost leads to the drop of both firms' profit. A simple rationale can be applied to explain why prices and profit drops in the centralized case.

As the retailer's ordering cost A_r increase, under the decentralized scenario, the retailer's ordering cycle T^{d*} increase without doubt. The vendor's ordering cycle $T_v^{d*}(T^{d*} \cdot n^{d*})$ increases in A_r. The prices p_v^{d*}, p_r^{d*} in the two channels increase, while w^{d*} decreases in parameter A_r. This is because for a higher A_r, the retailer's ordering frequency n^{d*} drops, which leads to the rise of the vendor's inventory holding costs. To encourage the retailer to order more products, the vendor offers a lower wholesale price. All these leads to the drop of the total profit, vendor's and retailer's profit under higher value of A_r. In the centralized case, the A_r is an internal operating cost, which has small impacts to the pricing decisions, but has negative impacts to the total profit.

For a higher procurement cost c_v, in the decentralized and the centralized case, the prices p_v^{d*}, p_r^{d*}, w^{d*} and ordering cycle increase T^{d*}, while total profit TP_{sc}^{d*}, vendor's profit $TP_v^{d*}$and retailer's TP_r^{d*} profit drop. In the centralized case, pricesp_v^{c*}, p_r^{c*}, w^{c*}, ordering cycle T^{c*} increases, while the total profit TP_{sc}^{c*} decreases in c_v. It is because, when procurement cost increases, firms need to set higher selling prices, which leads to the drop of market demand. Also, to reduce its inventory cost, the vendor should set a longer ordering cycle.

Tab. 9 Optimal centralized and decentralized decisions for different h_v, h_r, A_v, A_r and c_v

		Centralized Supply Chain					Decentralized supply chain							
		p_v^{c*}	p_r^{c*}	T^{c*}	n^{c*}	TP^{c*}	p_v^{d*}	w^{d*}	p_r^{d*}	T^{d*}	n^{d*}	TP_v^{d*}	TP_r^{d*}	TP^{d*}
	Default	10.99	11.22	2.92	10	562.34	11.11	10.89	13.29	2.69	12	458.46	30.15	488.62
h_v	-40%	10.88	11.14	2.90	11	610.41	10.99	10.81	13.24	2.73	13	505.14	31.37	536.52
	-20%	10.93	11.18	3.03	10	586.47	11.03	10.82	13.25	2.73	12	481.02	32.01	513.02
	20%	11.03	11.26	3.11	9	541.31	11.130	10.89	13.30	2.73	11	436.82	31.30	468.12
	40%	11.08	11.29	3.01	9	519.47	11.20	10.95	13.33	2.68	11	416.60	29.62	446.22
h_r	-40%	10.98	11.17	4.12	7	577.79	11.13	10.93	13.25	2.84	11	461.95	34.97	496.92
	-20%	10.98	11.20	3.23	9	568.61	11.12	10.90	13.29	2.73	12	460.07	31.08	491.15
	20%	10.99	11.24	2.90	10	558.27	11.10	10.86	13.30	2.66	12	456.77	29.24	486.00
	40%	11.00	11.26	2.65	11	552.90	11.09	10.83	13.31	2.65	12	455.00	28.34	483.34
A_v	-40%	10.82	11.03	2.81	8	690.56	10.91	10.64	13.17	3.09	8	570.93	42.61	613.54
	-20%	10.91	11.13	2.89	9	622.34	11.01	10.77	13.231	2.86	10	511.13	36.03	547.16
	20%	11.06	11.31	3.19	10	509.89	11.20	11.00	13.35	2.56	14	410.85	24.82	435.67
	40%	11.13	11.38	3.15	11	460.26	11.26	11.06	13.39	2.52	15	367.21	22.33	389.55
A_r	-40%	10.97	11.19	2.60	11	575.50	11.09	10.99	13.26	1.67	19	461.14	33.29	494.43
	-20%	10.98	11.21	2.88	10	569.24	11.08	10.91	13.26	2.23	14	459.59	33.15	492.75
	20%	10.99	11.23	3.25	9	557.89	11.10	10.82	13.30	3.21	10	457.16	29.46	486.62
	40%	11.00	11.24	3.28	9	551.76	11.11	10.79	13.31	3.62	9	456.01	27.16	483.17
c_v	-40%	10.03	10.30	2.86	10	875.83	10.13	9.99	12.69	2.41	13	721.70	52.44	774.14
	-20%	10.51	10.75	2.88	10	713.06	10.64	10.47	13.01	2.52	13	584.76	39.38	624.14
	20%	11.47	11.69	2.97	10	423.78	11.57	11.31	13.58	2.91	11	342.26	21.92	364.18
	40%	11.96	12.18	3.36	9	299.38	12.12	11.80	13.91	3.10	11	236.37	11.96	248.33

5 Supply chain coordination

In this section, a revenue sharing and two part tariff contract is utilized to coordinate the supply chain. When adopting the contract, the retailer commits to share a proportion of β of its revenue with the vendor and the vendor sets a lower wholesale price w. Then, the two firms negotiate on the lump sum fee F based on their powers. Use superscript $(\cdot)^{co}$ to denote the parameters under supply chain coordination.

5.1 Supply chain coordination with a revenue sharing and two part tariff contract

The unit time total profits of the retailer and the vendor are respectively given as

$$\begin{aligned} TP_r^{co} &= \frac{1}{nT}[(1-\beta)SR_r - PC_r - HC_r - OC_r] \\ &= \frac{1}{nT}\{(1-\beta)p_r d_r \frac{1-e^{-\mu nT}}{\mu} - h_r \frac{d_r}{\theta-\mu}\left(\frac{e^{\theta T}-1}{\theta} - \frac{e^{\mu T}-1}{\mu}\right)\frac{1-e^{-\mu nT}}{e^{\mu T}-1} \\ &-w\frac{d_r}{\theta-\mu}(e^{(\theta-\mu)T}-1)\frac{1-e^{-\mu nT}}{1-e^{-\mu T}} - nA_r\} + F. \end{aligned} \tag{33}$$

$$TP_v^{co} = \frac{1}{nT}[\beta SR_r + SR_v + WR_v - HC_v - PC_v - OC_v]$$

$$
\begin{aligned}
&= \frac{1}{nT}\{\beta p_r d_r \frac{1-e^{-\mu nT}}{\mu} + p_v D_v nT + w\frac{d_r}{\theta-\mu}(e^{(\theta-\mu)T}-1)\frac{1-e^{-\mu nT}}{1-e^{-\mu T}} \\
&-h_v D_v \frac{e^{\theta nT}-\theta nT-1}{\theta^2} - h_v\frac{d_r}{(\theta-\mu)\theta}\left(e^{(\theta-\mu)nT}(e^{-\theta nT}-1)+(e^{\theta T}-1)\frac{1-e^{-\mu nT}}{e^{\mu T}-1}\right) \\
&-c_v\frac{D_v}{\theta}(e^{\theta nT}-1) - c_v\frac{d_r}{\theta-\mu}(e^{(\theta-\mu)nT}-1) - A_v\} - F. \qquad (34)
\end{aligned}
$$

In the above two functions, it is shown that when F is negative, the lump sum fee is transferred from the retailer to the vendor. The following proposition characterizes the conditions that a revenue sharing and two part tariff contract can coordinate the decentralized supply chain.

Proposition 3 *When θ and μ are relatively small, the supply chain coordination can be achieved through a revenue sharing and two part tariff contract only if the mechanism (β^{co}, w^{co}, F) satisfies*

$$
\beta^{co} = 1 - \frac{b(2-(\theta-\mu)n^cT^c)(h_r d_r^c - 2A_r/T^{c2}) + d_r^c h_r(\theta-\mu)bn^cT^c}{(d_r^c - bp_r^c)d_r^c(\mu n^cT^c-2)(\theta-\mu)n^c + p_r^c d_r^c b((\theta-\mu)n^cT^c-2)\mu n^c} \in [0,1], \qquad (35)
$$

$$
w^{co} = \frac{(d_r^c - bp_r^c)(2-\mu n^cT^c)(2A_r/T^{c2} - h_r d_r^c) + p_r^c d_r^c b h_r T^c \mu n^c}{(d_r^c - bp_r^c)d_r^c(\mu n^cT^c-2)(\theta-\mu)n^c + p_r^c d_r^c b((\theta-\mu)n^cT^c-2)\mu n^c} > 0, \qquad (36)
$$

$$
F \in \{TP_r^{co}(\beta^{co}, w^{co}, F) \geqslant TP_r^{d*}, TP_v^{co}(\beta^{co}, w^{co}, F) \geqslant TP_v^{d*}\}, \qquad (37)
$$

in which $d_r^c = (1-\alpha)a - bp_r^c + rp_v^c$,
p_v^c, p_r^c, n^c, T^c are the optimal decisions in the centralized model,

According to the contract structure, three coordinating tools can be used by both players to establish a efficient solution, i.e., the wholesale price, the revenue sharing rate, and the lump sum fee. In this contract, F^{co} is in the range $[\overline{F}, \underline{F}]$, which allows the retailer to earn no less profit than that available in the decentralized model (TP_r^{d*}). Consider TP_r^{d*} as the retailer's reservation profit-the lowest profit level at which the retailer can accept the contract. (33) and (34) show that higher F^{co} benefits the retailer, whereas lower F^{co} benefits the vendor. Negotiating the value of F^{co} depends heavily on the bargain power of the retailer and the vendor in the supply chain.

5.2 The optimal lump sum fee

Nash bargain method can be applied to determine the optimal F, which can achieve the Pareto improvement. According to Baron et al. (2016), Nash bargain function $B(F)$ is modeled as follows

$$
B(F) = [TP_v^{co}(\beta^{co}, w^{co}, F) - TP_v^{d*}]^{\gamma}[TP_r^{co}(\beta^{co}, w^{co}, F) - TP_r^{d*}]^{1-\gamma}. \qquad (38)
$$

The goal of the negotiation is to maximize Nash bargain function $B(F)$ by finding an optimal F. In the function, parameter $\gamma, (\gamma \in [0,1])$ denotes the vendor's bargain power. The optimal F can be obtained by solving the equation $\frac{\partial B(F)}{\partial F} = 0$, which is shown in Proposition 4.

Proposition 4 *For fixed γ, the optimal lump sum fee is*

$$F^{co} = [TP_v^{co}(\beta^{co}, w^{co}, F = 0) - TP_v^{d*}](1-\gamma) - [TP_r^{co}(\beta^{co}, w^{co}, F = 0) - TP_r^{d*}]\gamma. \tag{39}$$

Proposition 4 shows that the optimal lump sum fee is decreasing in the vendor's bargain power.

5.3 An example

The objective of this subsection is to gain further insights of the coordination strategy through numerical tests. The parameters in the base model are also used in this subsection. Substituting the parameters into equations (35) and (36), the coordinating revenue sharing rate can be obtained as $\beta^{co} = 0.698$, the wholesale price is $w^{co} = 1.86\$/\text{unit}$. The profit of the retailer under coordination is $TP_r^{co} = 50.82\$ + F$, and the vendor's profit is $TP_v^{co} = 511.52\$ - F$. The profit of the vendor, the retailer and the supply chain with and without coordination are plotted in **Fig.3** with respect to F. Considering the condition in equation (37), by choosing a proper value of F in the range $F \in [-20.66, 53.06]$, the coordinated supply chain can reach Pareto improving. For a fixed γ, the optimal lump sum fee is $F^{co} = 50.06 - 73.72\gamma$. In this example, the negativity of F^{co} means the lump sum fee is transferred from the retailer to the vendor.

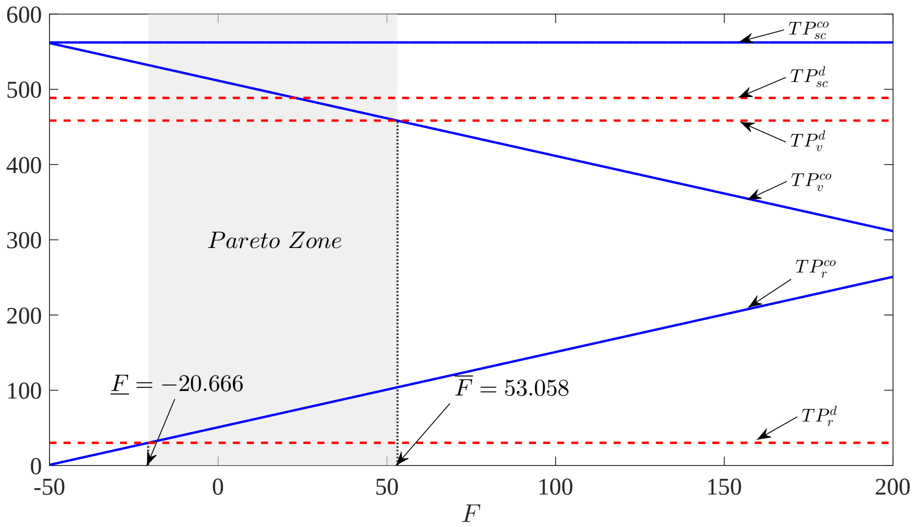
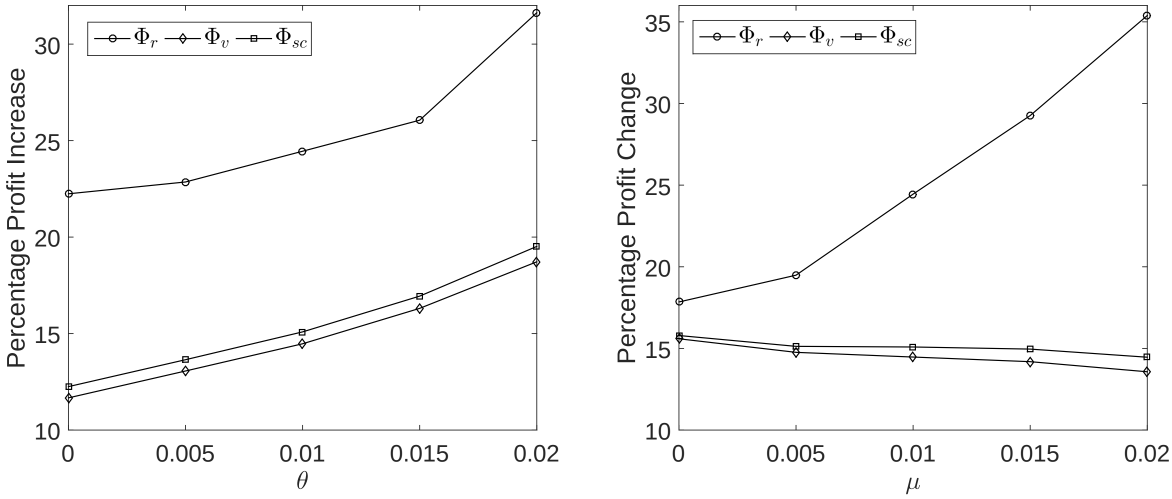


Fig. 3 Change of vendor, retailer and supply chain's profit with respect to F before and after coordination

The impacts of parameters θ, μ on the efficiency of the coordinating contract under fixed γ are studied. $\Phi_i = \frac{TP_i^{co} - TP_i^d}{TP_i^d} \times 100\%$ denotes the *Percentage Profit Increase* of $i, (i = v, r, sc)$, which also captures the coordination efficiency of the contract. Fig.4 (a) and (b) depict Φ_i with respect to parameters θ and μ.

It is shown in **Fig.4(a)**, for the vendor, the retailer and the supply chain, the coordination efficiency increases in the product deteriorating rate θ. It means that, supply chain members are more willing to coordinate with each other when deterioration rate is high. Comparing Φ_v and Φ_r, the retailer will benefit more from coordination. It is depicted in **Fig.4(b)** that when the quality of the product drops more fast, the coordination efficiency for the retailer rises rapidly. However, the coordination efficiency of the vendor



(a) Percentage profit increase with θ

(b) Percentage profit change with μ

Fig. 4 Percentage profit increase with respect to (a) θ and (b) μ when $\gamma = 0.9$

drops slightly. In another word, when quality drops more fast, the retailer is more willing to coordinate with the vendor. But quality dropping rate has no significant impacts to the vendor's coordinating efficiency.

6 Extension: delivery time as a decision variable

In this section, the direct channel delivery time, i.e., L, is assumed to be an endogenous parameter. In real life, when the direct channel delivery time is longer, more customers will switch from the direct channel to the retail channel. Following Hua et al. (2010), it is assumed that the direct channel demand is decreasing in L, while the retail channel demand is increasing in L. Then, the demand functions for the two channels can be formulated as

$$D_v^L = \alpha a - bp_v + rp_r - s_1 L, \tag{40}$$

$$D_r^L = [(1-\alpha)a - bp_r + rp_v + s_2 L]e^{-\mu t}, t \in [0, nT]. \tag{41}$$

in which, s_1 and s_2 are the lead time sensitivity of the demands in the direct and retail channel, respectively. Here, the delivery time L is controllable, which incurs an investment cost $c_L(L)$. Assuming the cost $c_L(L)$ is decreasing and convex in L, which follows the law of *Diminishing Marginal Utility*. Similar to Hua et al. (2010), the cost is formulated as $c_L(L) = \frac{s_3}{L+s_4}$. Note $[(1-\alpha)a - bp_r + rp_v + s_2 L]$ as d_r^L in equation (41). Under the consideration of endogenous delivery time, the two firms' profit can be formulated as

$$\begin{aligned} TP_r^L(p_r, T) &= \frac{1}{nT}\{p_r d_r^L \frac{1-e^{-\mu nT}}{\mu} - h_r \frac{d_r^L}{\theta - \mu}\left(\frac{e^{\theta T}-1}{\theta} - \frac{e^{\mu T}-1}{\mu}\right)\frac{1-e^{-\mu nT}}{e^{\mu T}-1} \\ &\quad - w\frac{d_r^L}{\theta-\mu}(e^{(\theta-\mu)T}-1)\frac{1-e^{-\mu nT}}{1-e^{-\mu T}} - nA_r\}. \end{aligned} \tag{42}$$

$$\begin{aligned} TP_v^L(p_v, w, n, L) &= \frac{1}{nT}\{p_v D_v^L nT + w\frac{d_r^L}{\theta-\mu}(e^{(\theta-\mu)T}-1)\frac{1-e^{-\mu nT}}{1-e^{-\mu T}} \\ &\quad - h_v D_v^L \frac{e^{\theta nT} - \theta nT - 1}{\theta^2} - h_v \frac{d_r^L}{(\theta-\mu)\theta}\left(e^{(\theta-\mu)nT}(1-e^{-\theta nT}) + (e^{\theta T}-1)\frac{1-e^{-\mu nT}}{1-e^{\mu T}}\right) \end{aligned}$$

$$-c_v \frac{D_v^L}{\theta}(e^{\theta nT} - 1) - c_v \frac{d_r^L}{\theta - \mu}(e^{(\theta-\mu)nT} - 1) - A_v\} - c_L(L). \tag{43}$$

In the centralized model, the two firms make decision together to maximize the total profit by deciding optimal p_v, p_r, T, n and L. In the decentralized model, as the leader, the vendor first determines p_v, w, n and L; then, the retailer sets p_r and T optimally based on the vendor's strategies. Considering endogenous direct channel delivery time makes the model more complex and it is hard to obtain the analytical results. Numerical tests are conducted to find some important managerial insights. Values for parameters of s_1, s_2, s_3 and s_4 are set as 10, 5, 100 and 1, respectively. Other parameters are set as that in the base model. For constant L, the optimal pricing decisions and inventory decisions in the centralized and decentralized models can be obtained by utilizing Algorithm 1 and 2, respectively. Then, with iteration of parameter L, the optimal delivery time can be found.

It is shown in **Fig.5** that, in the centralized supply chain, the total profit is concave in L and the optimal point is $L^{c*} = 0.86$ day. Then, in the decentralized model, the vendor's profit is also concave in L and the optimal point is $L^{d*} = 0.61$ day. It is found that, in the decentralized supply chain, the vendor should invest more to shorten the delivery time comparing to the centralized model, which conforms to the conclusions in Hua et al. (2010).

The sensitivity analysis of optimal delivery time in the centralized and decentralized models with respect to θ and μ are also conducted, which are shown in **Fig.6 (a)-(b)**. When deterioration rate (θ) rises, the vendor will invest less in reducing the delivery time in both the decentralized and centralized models. This is because shorter delivery time contributes to the total demand rate, while, it also results in the deterioration of more products. Thus, to avoid the deterioration cost, the vendor has less incentives to reduce the delivery lead time when θ is higher. When the quality dropping rate (μ) is higher, in both models, vendor will invest more to achieve a shorter delivery time. This is because higher μ results in quicker shrink of retail channel demand. To decelerate the demand dropping, the vendor should invest more on the delivery time and attract more customers.

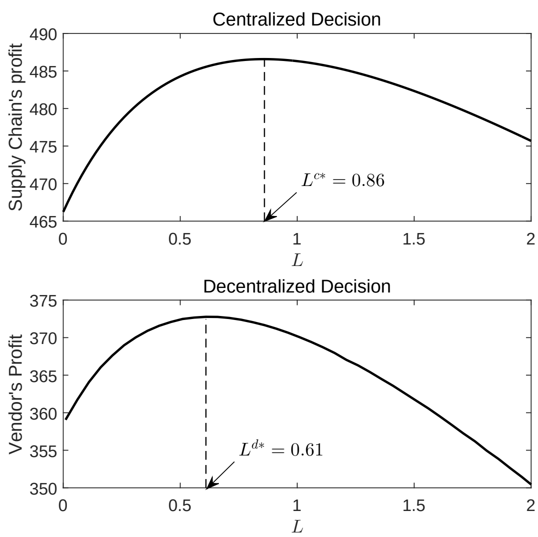
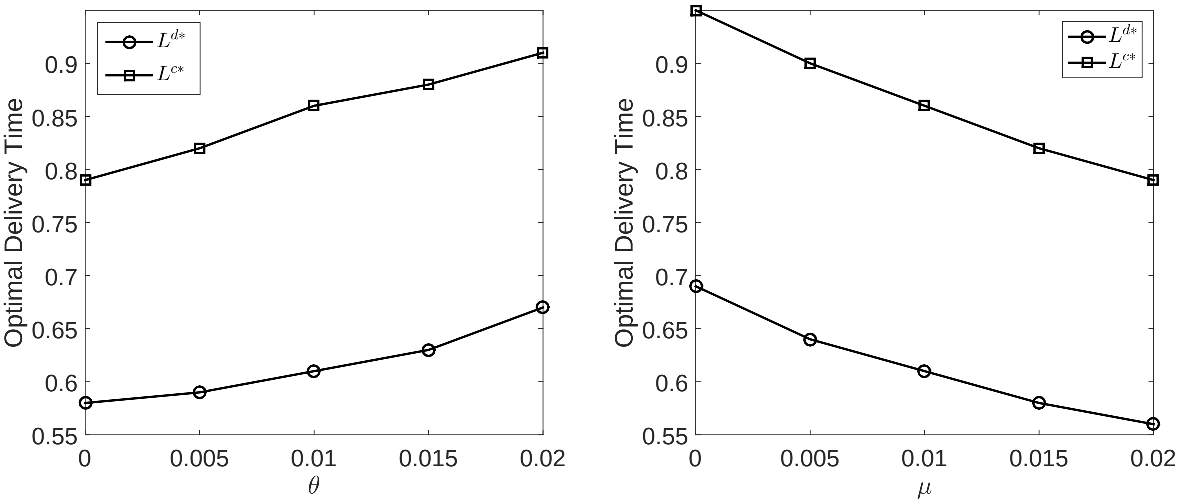


Fig. 5 Profit change with respect to L in the centralized and decentralized supply chain



(a) Optimal delivery time change with θ

(b) Optimal delivery time change with μ

Fig. 6 Optimal delivery time in centralized and decentralized models with respect to (a) θ and (b) μ

7 Conclusions

Nowadays, dual-channel supply chain structure, i.e., a traditional retailer channel and an online direct channel, is widely adopted by some companies selling deteriorating products. However, few literature considers about the deterioration property of products in dual-channel supply chain decisions. To fill this gap, centralized and decentralized models of a dual-channel supply chain for deterioration products with a vendor and a retailer are established. The two firms make decisions on pricing and inventory under channel competition and product deterioration. In the centralized model, the vendor and the retailer make decisions together to maximize the total profit of the supply chain. In the decentralized model, the vendor and retailer competes with each other and follows a Stackelberg gaming sequence in which the vendor acts as the leader. Due to the complexity of the models, no explicit solution can be obtained by calculation. Thus, two algorithms are proposed to solve the models.

The key findings are obtained through model analysis and numerical tests. The impacts of critical parameters (deterioration rate, competition intensity, inventory holding cost, ordering cost, etc.) to the price and inventory decisions are presented, from which, some interesting and meaningful results are summarized. In addition, it is found that a revenue sharing and two part tariff contract can coordinate the supply chain. Under applying the coordination contract, the total waste rate of the supply chain declines. In the extension, the optimal direct channel delivery time is studied. Results show that the vendor will invest more in reducing the delivery lead time in the decentralized model than in the centralized model. In addition, in both models, vendor will set a higher delivery lead time when deterioration rate is higher or quality dropping rate is smaller.

There are still some limitations of this paper. Firstly, in this paper, the deterioration rate does not change with time. However, in some cases, deterioration rate is not a constant, and it often changes with time. For example, Qin et al. (2014) and Chauhan & Singh (2015) assumed that the deterioration rate is increasing in time. Skouri et al. (2009) studied an inventory model with Weibull deterioration rate. Shah et al. (2013) studied a model with non-instantaneously deteriorating products with time dependent deterioration rate. In the future research, this study can be extended by incorporating time dependent deterioration rate. Secondly, the deterioration cost is not considered in this paper. However, in some conditions, companies also need to pay when dealing with the wastes (Zhang et al., 2015; Qin et al., 2014). In the future, deteriorating cost will be introduced to the model. Thirdly, it is assumed that only retail channel customers are sensitive to the products' quality because they can touch and feel the quality before their purchase. This assumption can also be relaxed by assuming both channel's customers are sensitive to products' quality in future study. Fourthly, in this study, the vendor is the Stackelberg leader in the dual-channel supply chain. In future research, models with a leading retailer can also be studied.

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Appendix

A Proof of Lemma 1

Proof The inventory level in the nth phase in time interval $t \in [(n-1)T, nT]$ satisfies the differential equation $\dot{I}_{vn}(t) = -\theta I_{vn}(t) - D_v, t \in [(n-1)T, nT]$ with boundary condition $I_{vn}(t = nT) = 0$.

Solving the equation, the inventory level in the nth phase is

$$I_{vn}(t) = \frac{D_v}{\theta}(e^{\theta(nT-t)} - 1), t \in [(n-1)T, nT]. \tag{44}$$

Then, solving the differential equation in the $(n-1)$th phase, i.e., $\dot{I}_{v(n-1)}(t) = -\theta I_{v(n-1)}(t) - D_v, t \in [(n-2)T, (n-1)T]$ with boundary condition $I_{v(n-1)}[t = (n-1)T] - I_{vn}[t = (n-1)T] = Q_{rn}$, the inventory level in time interval $t \in [(n-2)T, (n-1)T]$ can be solved as

$$I_{v(n-1)}(t) = \frac{D_v}{\theta}(e^{\theta(nT-t)} - 1) + Q_{rn}e^{\theta((n-1)T-t)}, t \in [(n-2)T, (n-1)T]. \tag{45}$$

Following the same approach, the inventory level in time interval $t \in [(n-3)T, (n-2)T]$ is

$$I_{v(n-2)}(t) = \frac{D_v}{\theta}(e^{\theta(nT-t)} - 1) + Q_{rn}e^{\theta(n-1)T-t} + Q_{r(n-1)}e^{\theta((n-2)T-t)}, t \in [(n-3)T, (n-2)T]. \tag{46}$$

Finally, the inventory level in time interval $t \in [0, T]$ is

$$I_{v1}(t) = \frac{D_v}{\theta}(e^{\theta(nT-t)} - 1) + Q_{rn}e^{\theta((n-1)T-t)} + Q_{r(n-1)}e^{\theta((n-2)T-t)} + ... + Q_{r2}e^{\theta(T-t)}, t \in [0, T]. \tag{47}$$

Based on the above analysis, the inventory level can be inducted as

$$I_{vj}(t) = \frac{D_v}{\theta}(e^{\theta(nT-t)} - 1) + \frac{d_r}{\theta - \mu}(e^{(\theta-\mu)nT} - e^{(\theta-\mu)jT})e^{-\theta t}, t \in [(j-1)T, jT], j = 1, 2, ..., n. \tag{48}$$

This ends the proof of Lemma 1. □

B Proof of Proposition 1

Proof Before the proof, some definitions are made:

$X_1 = \frac{1-e^{-\mu nT}}{\mu nT}$, $X_2 = \frac{e^{\theta nT} - \theta nT - 1}{\theta^2 nT}$, $X_3 = \frac{1}{(\theta-\mu)\theta nT}\left(e^{(\theta-\mu)nT}(1 - e^{-\theta nT}) + (e^{\theta T} - 1)\frac{1-e^{-\mu nT}}{1-e^{\mu T}}\right)$,

$X_4 = \frac{1}{(\theta-\mu)nT}\left(\frac{e^{\theta T}-1}{\theta} - \frac{e^{\mu T}-1}{\mu}\right)\frac{1-e^{-\mu nT}}{e^{\mu T}-1}$, $\quad X_5 = \frac{1}{\theta nT}(e^{\theta nT}-1)$, $\quad X_6 = \frac{1}{(\theta-\mu)nT}(e^{(\theta-\mu)nT}-1)$,
$X_7 = \frac{1}{(\theta-\mu)nT}(e^{(\theta-\mu)T}-1)\frac{1-e^{-\mu nT}}{1-e^{-\mu T}}$.

The profit function can be rewritten as

$$TP_{sc} = p_v D_v + p_r d_r X_1 - h_v D_v X_2 - h_v d_r X_3 - h_r d_r X_4 - c_v D_v X_5 - c_v d_r X_6 - \frac{A_r}{T} - \frac{A_v}{nT}. \tag{49}$$

(1)When n and T are fixed, taking the second order derivative of TP_{sc} with respect to p_v and p_r , the Hessian matrix can be obtained as

$$H = \begin{pmatrix} \frac{\partial^2 TP_{sc}}{\partial p_v^2} & \frac{\partial^2 TP_{sc}}{\partial p_v \partial p_r} \\ \frac{\partial^2 TP_{sc}}{\partial p_r \partial p_v} & \frac{\partial^2 TP_{sc}}{\partial p_r^2} \end{pmatrix} = \begin{pmatrix} -2b & (1+X_1)r \\ (1+X_1)r & -2b \end{pmatrix}. \tag{50}$$

When the quality losing rate μ is not very large and $b > r$, $|H| = 4b^2 - (1+X_1)^2 r^2 > 0$ is satisfied. Also $\frac{\partial^2 TP_{sc}}{\partial p_v^2} = -2b < 0$, so TP_{sc} is jointly concave in p_v and p_r. When equating both $\frac{\partial TP_{sc}}{\partial p_v}$ and $\frac{\partial TP_{sc}}{\partial p_r}$ to zero, the optimal prices can be obtained by solving the equation set, which can be expressed as

$$P_v^{c*} = \frac{B_2 r + 2bB_1X_1 + rX_1B_2 - arX_1 - arX_1^2 + \alpha arX_1^2 - 2\alpha abX_1 + \alpha arX_1}{r^2X_1^2 - 4b^2X_1 + 2r^2X_1 + r^2}, \tag{51}$$

$$P_r^{c*} = \frac{2bB_2 + rB_1 + rB_1X_1 - 2abX_1 - \alpha ar + 2\alpha abX_1 - \alpha arX_1}{r^2X_1^2 - 4b^2X_1 + 2r^2X_1 + r^2}, \tag{52}$$

in which $B_1 = -h_v bX_2 + h_v rX_3 + h_r rX_4 - c_v bX_5 + c_v rX_6$, $B_2 = h_v rX_2 - h_v bX_3 - h_r bX_4 + c_v rX_5 - c_v bX_6$.

(2)For fixed p_v,p_r and n, there is only one decision parameter T.

Taking the second derivative of the profit function with respect to T, we have

$$\frac{\partial^2 TP_{sc}}{\partial T^2} = p_r d_r X_1^{''} - h_v D_v X_2^{''} - h_v d_r X_3^{''} - h_r d_r X_4^{''} - c_v D_v X_5^{''} - c_v d_r X_6^{''} - \frac{2A_v}{nT^3} - \frac{2A_r}{T^3}. \tag{53}$$

For $X_1^{''} = \frac{e^{-\mu nT}}{T} + \frac{e^{-\mu nT}-1}{\mu nT^2}$. When setting $x = -\mu nT$, $X_1^{''} = \frac{xe^{-x}+e^{-x}-1}{\mu nT^2}$. Defining a new function $F(x) = xe^{-x} + e^{-x} - 1$. When $x \to 0$, $X_1^{''} = 0$. The first derivative of $F(x)$ satisfies $F^{'}(x) = -xe^{-x} < 0$. Thus in the interval $T \in (0, +\infty)$,$X_1^{''} < 0$ holds.

For $X_2^{''} = \frac{1}{\theta^2 nT^3}(\theta^2 n^2 T^2 e^{\theta nT} - 2\theta nTe^{\theta nT} + 2e^{\theta nT} - 2)$, set $x = \theta nT$ and define a new function $F(x) = x^2e^x - 2xe^x + 2e^x - 2$ in which $x \in (0, +\infty)$. When $x \to 0$, $F(x) = 0$ and the first derivative satisfies $F^{'}(x) = x^2e^x > 0$. Thus, in the interval $T \in (0, +\infty)$, $X_2^{''} > 0$ always holds.

The proofs of $X_i^{''} > 0$, $(i = 3, 4, 5, 6)$ are the same as $X_2^{''} > 0$. Finally, the second derivative of profit function satisfies $\frac{\partial^2 TP_{sc}}{\partial T^2} < 0$, and TP_{sc} is concave in T.

(3) Taking the second derivative of the profit function with respect to T, we have

$$\frac{\partial^2 TP_{sc}}{\partial n^2} = p_r d_r X_1^{''} - h_v D_v X_2^{''} - h_v d_r X_3^{''} - h_r d_r X_4^{''} - c_v D_v X_5^{''} - c_v d_r X_6^{''} - \frac{2A_v}{n^3 T}. \tag{54}$$

For $X_1^{''} = \frac{e^{-\mu nT}}{n} + \frac{e^{-\mu nT}-1}{\mu n^2 T}$. When setting $x = -\mu nT$, $X_1^{''} = \frac{xe^{-x}+e^{-x}-1}{\mu nT^2}$. Defining a new function $F(x) = xe^{-x} + e^{-x} - 1$. When $x \to 0$, $X_1^{''} = 0$. The first derivative of $F(x)$ satisfies $F^{'}(x) = -xe^{-x} < 0$. Thus in the interval $T \in (0, +\infty)$,$X_1^{''} < 0$ holds.

For $X_2^{''} = \frac{1}{\theta^2 n^3 T}(\theta^2 n^2 T^2 e^{\theta nT} - 2\theta nTe^{\theta nT} + 2e^{\theta nT} - 2)$, set $x = \theta nT$ and define a new function $F(x) = x^2e^x - 2xe^x + 2e^x - 2$, for $x \in (0, +\infty)$. When $x \to 0$, $F(x) = 0$ and the first derivative satisfies $F^{'}(x) = x^2e^x > 0$. Thus in the interval $n \in (0, +\infty)$,$X_2^{''} > 0$ holds.

The proofs of $X_i^{''} > 0$, $i = 3, 4, 5, 6$ are the same as $X_2^{''} > 0$. Finally, the second derivative of profit function satisfies $\frac{\partial^2 TP_{sc}}{\partial n^2} < 0$, and TP_{sc} is concave in n.

In the above analysis, we proved the existence and uniqueness for the decision variables. To obtain the analytical results, we approximate the exponential terms and solve equations $\frac{\partial TP_{sc}}{\partial n} = 0$ and $\frac{\partial TP_{sc}}{\partial n} = 0$, respectively. We use the Taylor expansion to approximate exponential terms. For example, the term $e^{\theta T} - 1$ is approximated to $e^{\theta T} - 1 \approx 1 + \theta T + \frac{\theta^2 T^2}{2} - 1 = \theta T + \frac{\theta^2 T^2}{2}$. Thus, we obtain the final results in Proposition 1.

This ends the proof of Proposition 1. □

C Proof of Proposition 2

Proof The vendor's profit function can be expressed as

$$TP_v = p_v D_v + w d_r X_7 - h_v D_v X_2 - h_v d_r X_3 - c_v D_v X_5 - c_v d_r X_6 - \frac{A_v}{nT}. \tag{55}$$

The retailer's profit function can be expressed as

$$TP_r = p_r d_r X_1 - h_r d_r X_4 - w d_r X_7 - \frac{A_r}{T}. \tag{56}$$

(1)When T and n are fixed, the value of $X_i, (i = 1, 2, 3, 4, 5, 6, 7)$ are determined. Equating the first derivative of retailer's profit function to zero, the optimal price can be derived as

$$p_r(p_v, w) = \frac{1}{2b}((1-\alpha)a + rp_v + bh_r X_4 + bX_7 w). \tag{57}$$

Substitute it into demand function, we have

$$D_v(p_v, w) = (\frac{r(1-\alpha)}{2b} + \alpha)a + (\frac{r^2}{2b} - b)p_v + \frac{1}{2} r h_r X_4(T) + \frac{1}{2} r w X_7(T), \tag{58}$$

$$d_r(p_v, w) = \frac{(1-\alpha)a}{2} + \frac{1}{2} r p_v - \frac{1}{2} b h_r X_4 - \frac{1}{2} b w X_7. \tag{59}$$

Substitute the demand functions into vendor's profit function, and take the first derivative of TP_v with respect to p_v and w, there is $\frac{\partial^2 TP_v}{\partial w^2} = -bX_7^2$, $\frac{\partial^2 TP_v}{\partial p_v^2} = \frac{r^2}{b} - 2b$, $\frac{\partial^2 TP_v}{\partial w \partial p_v} = \frac{\partial^2 TP_v}{\partial p_v \partial w} = rX_7$. The Hessian matrix is

$$H = \begin{pmatrix} \frac{\partial^2 TP_v}{\partial w^2} & \frac{\partial^2 TP_v}{\partial w \partial p_v} \\ \frac{\partial^2 TP_v}{\partial p_v \partial w} & \frac{\partial^2 TP_v}{\partial p_v^2} \end{pmatrix} = \begin{pmatrix} -bX_7^2 & rX_7 \\ rX_7 & \frac{r^2}{b} - 2b \end{pmatrix} \tag{60}$$

For $\frac{\partial^2 TP_v}{\partial w^2} = -bX_7^2 < 0$, $|H| = (b^2 - r^2)X_7^2 > 0$, vendor's profit function is jointly concave in p_v and w. And the optimal solution can be derived from the first order conditions.
$\frac{\partial TP_v(p_v, p_r(p_v,w),w)}{\partial p_v} = 0$, $\frac{\partial TP_v(p_v, p_r(p_v,w),w)}{\partial w} = 0$. Then the optimal prices can be determined as

$$p_v^{d*} = \frac{2bC_2 + 2rC_1 - 2\alpha ab - arX_7 + \alpha ar + \alpha arX_7}{2(r^2 X_7 - 2b^2 + r^2)}, \tag{61}$$

$$w^{d*} = \frac{2b^2 C_1 - r^2 C_1 - ab^2 X_7 + \alpha ab^2 X_7 + brX_7 C_2 - \alpha abrX_7}{X_7 b(r^2 X_7 - 2b^2 + r^2)}, \tag{62}$$

$$p_r^{d*} = \frac{1}{2b}((1-\alpha)a + rp_v^{d*} + bh_r X_4 + bX_7 w^{d*}), \tag{63}$$

in which $C_1 = \frac{h_r b X_4 X_7}{2} + \frac{h_v r X_2 X_7}{2} - \frac{h_v b X_3 X_7}{2} + \frac{c_v r X_5 X_7}{2} - \frac{c_v b X_6 X_7}{2}$, $C_2 = h_v X_2(\frac{r^2}{2b} - b) + \frac{h_v r X_3}{2} + c_v X_5(\frac{r^2}{2b} - b) + \frac{c_v X_6 r}{2} - \frac{h_r r X_4}{2}$

(2)Taking the second derivative of retailer's profit function with respect to T , we have

$$\frac{\partial^2 TP_r}{\partial T^2} = p_r d_r X_1^{''} - h_r d_r X_4^{''} - w d_r X_7^{''} - \frac{2A_r}{T^3}. \tag{64}$$

As proved in proposition 1, $X_1^{''} < 0$, $X_4^{''} > 0$ and $X_7^{''} > 0$ are satisfied. Thus $\frac{\partial^2 TP_r}{\partial T^2} < 0$, and retailer's profit function is concave in T.

(3)Taking the second derivative of vendor's profit function with respect to n ,

$$\frac{\partial^2 TP_v}{\partial n^2} = w d_r X_7^{''} - h_v D_v X_2^{''} - h_v d_r X_3^{''} - c_v D_v X_5^{''} - c_v d_r X_6^{''}(n) - \frac{2A_r}{n^3 T}. \tag{65}$$

As proved in proposition 1, $X_i^{''} > 0$ $.i = 2, 3, 5, 6$. For $X_7^{''} = \frac{1}{(\theta-\mu)nT}(e^{(\theta-\mu)T} - 1)\frac{-\mu^2 T^2 e^{-\mu nT}}{1-e^{-\mu T}} < 0$. Finally, the second derivative of retailer's profit function satisfies $\frac{\partial^2 TP_v}{\partial n^2} < 0$, thus TP_v is concave in n. Although is an integer variable, it is obvious that there exists a unique value of n that maximize the profit function when p_v, p_r and T are constant.

In the above analysis, we proved the existence and uniqueness for the decision variables. To obtain the analytical results, we approximate the exponential terms and solve equations $\frac{\partial TP_r}{\partial T} = 0$ and $\frac{\partial TP_v}{\partial n} = 0$, respectively. We use the Taylor expansion to approximate exponential terms. We obtain the final results in Proposition 2.
This ends the proof of Proposition 2. □

D Proof of Proposition 3

Proof For any given w, β and F, the retailer's retail price p_r and ordering cycle T should satisfy

$$\frac{\partial TP_r^{co}}{\partial p_r}|_{p_r=p_r^c} = 0, \tag{66}$$

$$\frac{\partial TP_r^{co}}{\partial T}|_{T=T^c} = 0. \tag{67}$$

Taking the first order partial derivative of (31) with respect to p_r and T and setting them to zero yields

$$\frac{\partial TP_r^{co}}{\partial p_r} = (1-\beta)(1-\mu nT/2)((1-\alpha)a - 2bp_r + rp_v) + h_r bT/2 + wb(1-(\theta-\mu)nT/2) = 0, \tag{68}$$

$$\frac{\partial TP_r^{co}}{\partial T} = (1-\beta)p_r d_r(-\mu n/2) - h_r d_r/2 - wd_r(-(\theta-\mu)n/2) + A_r/T^2 = 0. \tag{69}$$

Substituting $p_r = p_r^c$ and $T = T^c$ into the two equations and solving (β, w), the expressions of β^{co} and w^{co} in proposition 3 can be obtained. In addition, both parties' profit should be no less than that without coordination. Thus the lump sum fee need to satisfy

$$F \in \{TP_r^{co}(\beta^{co}, w^{co}, F) \geqslant TP_r^{d*}, TP_v^{co}(\beta^{co}, w^{co}, F) \geqslant TP_v^{d*}\}. \tag{70}$$

This ends the proof of Proposition 3. □

E Proof of Proposition 4

Proof Calculating the first and second order derivative of $B(F)$ as follows

$$\begin{aligned}\frac{\partial B(F)}{\partial F} = &\{(1-\gamma)[TP_v^{co}(\beta^{co}, w^{co}, F=0) - TP_v^{d*} - F] - \gamma[TP_r^{co}(\beta^{co}, w^{co}, F=0) - TP_r^{d*} + F]\} \times \\ &[TP_v^{co}(\beta^{co}, w^{co}, F=0) - TP_v^{d*} - F]^{\gamma-1}[TP_r^{co}(\beta^{co}, w^{co}, F=0) - TP_r^{d*}]^{-\gamma}.\end{aligned} \tag{71}$$

$$\begin{aligned}\frac{\partial^2 B(F)}{\partial F^2} = &-\gamma(1-\gamma)[TP_v^{co}(\beta^{co}, w^{co}, F=0) - TP_v^{d*} - F]^{\gamma-2}[TP_r^{co}(\beta^{co}, w^{co}, F=0) - TP_r^{d*} + F]^{-\gamma-1} \times \\ &[TP_v^{co}(\beta^{co}, w^{co}, F=0) - TP_v^{d*} + TP_r^{co}(\beta^{co}, w^{co}, F=0) - TP_r^{d*}]^2.\end{aligned} \tag{72}$$

The second order derivative $\frac{\partial^2 B(F)}{\partial F^2}$ is negative, which means there exists an optimal F that maximize the function. Equating $\frac{\partial B(F)}{\partial F}$ to zero, the optimal F can be obtained as

$$F^* = (TP_r^{co}(\beta^{co}, w^{co}, F=0) - TP_v^{d*}) * (1-\gamma) - (TP_r^{co}(\beta^{co}, w^{co}, F=0) - TP_r^{d*}) * \gamma. \tag{73}$$

This ends the proof of Proposition 4. □

References

Baron, O., Berman, O., & Wu, D. (2016). Bargaining within the supply chain and its implications in an industry. *Decision Sciences*, *47*, 193–218.

Batarfi, R., Jaber, M. Y., & Zanoni, S. (2016). Dual-channel supply chain: A strategy to maximize profit. *Applied Mathematical Modelling*, *40*, 9454–9473.

Burwell, T. H., Dave, D. S., Fitzpatrick, K. E., & Roy, M. R. (1997). Economic lot size model for price-dependent demand under quantity and freight discounts. *International Journal of Production Economics*, *48*, 141–155.

Cai, G. G. (2010). Channel selection and coordination in dual-channel supply chains. *Journal of Retailing*, *86*, 22–36.

Cai, G. G., Zhang, Z. G., & Zhang, M. (2009). Game theoretical perspectives on dual-channel supply chain competition with price discounts and pricing schemes. *International Journal of Production Economics*, *117*, 80–96.

Chauhan, A., & Singh, A. P. (2015). Optimal replenishment and ordering policy for time dependent demand and deterioration with discounted cash flow analysis. *International Journal of Mathematics in Operational Research*, *6*, 407–436.

Chen, J., Liang, L., Yao, D.-Q., & Sun, S. (2017). Price and quality decisions in dual-channel supply chains. *European Journal of Operational Research*, *269*, 935–948.

Chen, J., Zhang, H., & Sun, Y. (2012). Implementing coordination contracts in a manufacturer Stackelberg dual-channel supply chain. *Omega*, *40*, 571–583.

Chen, J.-M., & Chen, T.-H. (2007). The profit-maximization model for a multi-item distribution channel. *Transportation Research Part E: Logistics and Transportation Review*, *43*, 338–354.

Chen, T. H. (2015). Effects of the pricing and cooperative advertising policies in a two-echelon dual-channel supply chain. *Computers & Industrial Engineering*, *87*, 250–259.

Chiang, W. K., Chhajed, D., & Hess, J. D. (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management science*, *49*, 1–20.

Chiang, W. K., & Monahan, G. E. (2005). Managing inventories in a two-echelon dual-channel supply chain. *European Journal of Operational Research*, *162*, 325–341.

Choi, T.-M., Li, Y., & Xu, L. (2013). Channel leadership, performance and coordination in closed loop supply chains. *International Journal of Production Economics*, *146*, 371–380.

Dan, B., Xu, G., & Liu, C. (2012). Pricing policies in a dual-channel supply chain with retail services. *International Journal of Production Economics*, *139*, 312–320.

Dumrongsiri, A., Fan, M., Jain, A., & Moinzadeh, K. (2008). A supply chain model with direct and retail channels. *European Journal of Operational Research*, *187*, 691–718.

Dye, C.-Y., Chang, H.-J., & Teng, J.-T. (2006). A deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging. *European Journal of Operational Research*, *172*, 417–429.

Dye, C.-Y., & Hsieh, T.-P. (2012). An optimal replenishment policy for deteriorating items with effective investment in preservation technology. *European Journal of Operational Research*, *218*, 106–112.

Dye, C.-Y., & Hsieh, T.-P. (2013). A particle swarm optimization for solving lot-sizing problem with fluctuating demand and preservation technology cost under trade credit. *Journal of Global Optimization*, *55*, 655–679.

Fibich, G., Lowengart, O., & Gavious, A. (2003). Explicit solutions of optimization models and differential games with nonsmooth (asymmetric) reference-price effects. *Operations Research*, *51*, 721–734.

Ghare, P., & Schrader, G. (1963). A model for exponentially decaying inventory. *Journal of Industrial Engineering*, *14*, 238–243.

Giri, B. C., Jalan, A., & Chaudhuri, K. (2003). Economic order quantity model with Weibull deterioration distribution, shortage and ramp-type demand. *International Journal of Systems Science*, *34*, 237–243.

He, R., Xiong, Y., & Lin, Z. (2016). Carbon emissions in a dual channel closed loop supply chain: the impact of consumer free riding behavior. *Journal of Cleaner Production*, *134*, 384–394.

He, Y., Zhang, P., & Yao, Y. (2014). Unidirectional transshipment policies in a dual-channel supply chain. *Economic Modelling*, *40*, 259–268.

Hsu, P., Wee, H., & Teng, H. (2010). Preservation technology investment for deteriorating inventory. *International Journal of Production Economics*, *124*, 388–394.

Hua, G., Wang, S., & Cheng, T. E. (2010). Price and lead time decisions in dual-channel supply chains. *European journal of operational research*, *205*, 113–126.

Huang, S., Yang, C., & Zhang, X. (2012). Pricing and production decisions in dual-channel supply chains with demand disruptions. *Computers & Industrial Engineering*, *62*, 70–83.

Huang, W., & Swaminathan, J. M. (2009). Introduction of a second channel: Implications for pricing and profits. *European Journal of Operational Research*, *194*, 258–279.

Ji, J., Zhang, Z., & Yang, L. (2017). Carbon emission reduction decisions in the retail-/dual-channel supply chain with consumers' preference. *Journal of Cleaner Production*, *141*, 852–867.

Khuntonthong, P., Chakpitak, N., & Neubert, G. (2013). Management framework for a high-value of agricultural product to increase income for farmers in rural area. *KMITL-Science and Technology Journal*, *13*, 42–50.

Kopalle, P. K., Rao, A. G., & Assuno, J. L. (1996). Asymmetric reference price effects and dynamic pricing policies. *Marketing Science*, *15*, 60–85.

Lee, J. H., & Moon, I. K. (2006). Coordinated inventory models with compensation policy in a three level supply chain. In *Computational Science and Its Applications-ICCSA 2006* (pp. 600–609). Springer.

Li, B., Hou, P. W., Chen, P., & Li, Q. H. (2016a). Pricing strategy and coordination in a dual channel supply chain with a risk-averse retailer. *International Journal of Production Economics*, *178*, 154–168.

Li, B., Zhu, M., Jiang, Y., & Li, Z. (2016b). Pricing policies of a competitive dual-channel green supply chain. *Journal of Cleaner Production*, *112*, 2029–2042.

Li, Q.-H., & Li, B. (2016). Dual-channel supply chain equilibrium problems regarding retail services and fairness concerns. *Applied Mathematical Modelling*, *40*, 7349–7367.

Liang, Y., & Zhou, F. (2011). A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. *Applied Mathematical Modelling*, *35*, 2221–2231.

Lin, Y., Yu, J. C., & Wang, K.-J. (2009). An efficient replenishment model of deteriorating items for a supplier–buyer partnership in hi-tech industry. *Production Planning and Control*, *20*, 431–444.

Lin, Y.-H., Lin, C., & Lin, B. (2010). On conflict and cooperation in a two-echelon inventory model for deteriorating items. *Computers & Industrial Engineering*, *59*, 703–711.

Liu, B., Zhang, R., & Xiao, M. (2010). Joint decision on production and pricing for online dual channel supply chain system. *Applied Mathematical Modelling*, *34*, 4208–4218.

Liu, M., Cao, E., & Salifou, C. K. (2015). Pricing strategies of a dual-channel supply chain with risk aversion. *Transportation Research Part E: Logistics & Transportation Review*, *90*, 108–120.

Lo, S.-T., Wee, H.-M., & Huang, W.-C. (2007). An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation. *International Journal of Production Economics*, *106*, 248–260.

Lu, Q., & Liu, N. (2015). Effects of e-commerce channel entry in a two-echelon supply chain: A comparative analysis of single- and dual-channel distribution systems. *International Journal of Production Economics*, *165*, 100–111.

Mahata, G. C. (2012). An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. *Expert systems with Applications*, *39*, 3537–3550.

Matsui, K. (2015). Asymmetric product distribution between symmetric manufacturers using dual-channel supply chains. *European Journal of Operational Research*, *248*, 646–657.

Matsui, K. (2017). When should a manufacturer set its direct price and wholesale price in dual-channel supply chains? *European Journal of Operational Research*, *258*, 501–511.

Moss, C. B., Schmitz, T. G., Kagan, A., Schmitz, A. et al. (2003). Institutional economics and the emergence of E-commerce in agribusiness. *Journal of Agribusiness*, *21*, 83–102.

Mukhopadhyay, S. K., Yao, D.-Q., & Yue, X. (2008). Information sharing of value-adding retailer in a mixed channel hi-tech supply chain. *Journal of Business Research*, *61*, 950–958.

Noh, J. S., Kim, J. S., & Sarkar, B. (2016). Stochastic joint replenishment problem with quantity discounts and minimum order constraints. *Operational Research-An International Journal*, (pp. 1–28, DOI 10.1007/s12351–016–0281–6.).

Qin, Y., Wang, J., & Wei, C. (2014). Joint pricing and inventory control for fresh produce and foods with quality and physical quantity deteriorating simultaneously. *International Journal of Production Economics*, *152*, 42–48.

Rodriguez, B., & Aydin, G. (2015). Pricing and assortment decisions for a manufacturer selling through dual channels. *European Journal of Operational Research*, *242*, 901–909.

Sana, S., Goyal, S., & Chaudhuri, K. (2004). A production–inventory model for a deteriorating item with trended demand and shortages. *European Journal of Operational Research*, *157*, 357–371.

Sarkar, B. (2011). An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production. *Applied Mathematics and Computation*, *218*, 8295–8308.

Sarkar, B. (2012a). An EOQ model with delay in payments and time varying deterioration rate. *Mathematical and Computer Modelling*, *55*, 367–377.

Sarkar, B. (2012b). An inventory model with reliability in an imperfect production process. *Applied Mathematics and Computation*, *218*, 4881–4891.

Sarkar, B. (2013). A production-inventory model with probabilistic deterioration in two-echelon supply chain management. *Applied Mathematical Modelling*, *37*, 3138–3151.

Sarkar, B. (2016). Supply chain coordination with variable backorder, inspections, and discount policy for fixed lifetime products. *Mathematical Problems in Engineering*, *2016*, 1–14.

Sarkar, B., Majumder, A., Sarkar, M., Dey, B., & Roy, G. (2016). Two-echelon supply chain model with manufacturing quality improvement and setup cost reduction. *Journal of Industrial and Management Optimization*, *13*, 1085–1104.

Sarkar, B., Saren, S., & Cárdenas-Barrón, L. E. (2015). An inventory model with trade-credit policy and variable deterioration for fixed lifetime products. *Annals of Operations Research*, *229*, 677–702.

Sarkar, B., Saren, S., & Wee, H. M. (2013). An inventory model with variable demand, component cost and selling price for deteriorating items. *Economic Modelling*, *30*, 306–310.

Sarkar, M., & Sarkar, B. (2013). An economic manufacturing quantity model with probabilistic deterioration in a production system. *Economic Modelling*, *31*, 245–252.

Sarker, B. R., Mukherjee, S., & Balan, C. V. (1997). An order-level lot size inventory model with inventory-level dependent demand and deterioration. *International Journal of Production Economics*, *48*, 227–236.

Shah, N. H., Soni, H. N., & Patel, K. A. (2013). Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. *Omega*, *41*, 421–430.

Skouri, K., Konstantaras, I., Papachristos, S., & Ganas, I. (2009). Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. *European Journal of Operational Research*, *192*, 79–92.

Song, H.-m., & Zhao, Z.-q. (2010). A study of Stackelberg inventory model with vendor-buyer interaction on lead time. In *Management and Service Science (MASS), 2010 International Conference on* (pp. 1–4). IEEE.

Sorger, G. (1988). Reference price formation and optimal marketing strategies. *Optimal Control Theory & Economic Analysis*, *3*, 97–120.

Takahashi, K., Aoi, T., Hirotani, D., & Morikawa, K. (2011). Inventory control in a two-echelon dual-channel supply chain with setup of production and delivery. *International Journal of Production Economics*, *133*, 403–415.

Thangam, A., & Uthayakumar, R. (2009). Two-echelon trade credit financing for perishable items in a supply chain when demand depends on both selling price and credit period. *Computers & Industrial Engineering*, *57*, 773–786.

Tsay, A. A., & Agrawal, N. (2004). Channel conflict and coordination in the e-commerce age. *Production and Operations Management*, *13*, 93–110.

Wang, K.-J., Lin, Y., & Jonas, C. (2011). Optimizing inventory policy for products with time-sensitive deteriorating rates in a multi-echelon supply chain. *International Journal of Production Economics*, *130*, 66–76.

Wang, X., & Li, D. (2012). A dynamic product quality evaluation based pricing model for perishable food supply chains. *Omega*, *40*, 906–917.

Wilder, C. (1999). HP's online push. *Information Week*, *4*, 53–54.

Xiao, T., Choi, T.-M., & Cheng, T. (2014). Product variety and channel structure strategy for a retailer-stackelberg supply chain. *European Journal of Operational Research*, *233*, 114–124.

Xiao, T., & Shi, J. (2016). Pricing and supply priority in a dual-channel supply chain. *European Journal of Operational Research*, *254*, 813–823.

Xie, J. P., Liang, L., Liu, L. H., & Ieromonachou, P. (2017). Coordination contracts of dual-channel with cooperation advertising in closed-loop supply chains. *International Journal of Production Economics*, *183*, 528–538.

Xu, H., Liu, Z. Z., & Zhang, S. H. (2012). A strategic analysis of dual-channel supply chain design with price and delivery lead time considerations. *International Journal of Production Economics*, *139*, 654–663.

Yan, B., Wang, T., Liu, Y.-P., & Liu, Y. (2016). Decision analysis of retailer-dominated dual-channel supply chain considering cost misreporting. *International Journal of Production Economics*, *178*, 34–41.

Yan, R. (2008). Profit sharing and firm performance in the manufacturer-retailer dual-channel supply chain. *Electronic Commerce Research*, *8*, 155–172.

Yan, R., & Pei, Z. (2009). Retail services and firm profit in a dual-channel market. *Journal of retailing and consumer services*, *16*, 306–314.

Yang, J. Q., Zhang, X. M., Fu, H. Y., & Liu, C. (2017). Inventory competition in a dual-channel supply chain with delivery lead time consideration. *Applied Mathematical Modelling*, *42*, 675–692.

Yu, D.-Z., Cheong, T., & Sun, D. (2016). Impact of supply chain power and drop-shipping on a manufacturers optimal distribution channel strategy. *European Journal of Operational Research*, *259*, 554–563.

Yue, X., & Liu, J. (2006). Demand forecast sharing in a dual-channel supply chain. *European Journal of Operational Research*, *174*, 646–667.

Zhang, J., Liu, G., Zhang, Q., & Z, B. (2015). Coordinating a supply chain for deteriorating items with a revenue sharing and cooperative investment contract. *Omega*, *56*, 37–49.